Visual Fatigue and Motion Sickness Induced by 3D Video Clip

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It is generally understood that “accommodation” and “convergence” are mismatched during stereoscopic vision and that this is the main reason for the visual fatigue and the visually induced motion sickness (VIMS). The aim of this paper is to explain results of recent studies by the present authors on this problem. Fixation distances between accommodation and convergence in young and middle-aged subjects are compared while they viewed 2D and 3D video clips, and the severity of motion sickness induced by viewing 3D video clips on a liquid crystal display (LCD) was analyzed by comparing to that induced by viewing these films on a head-mounted display (HMD), and then the body sway models are discussed. It is concluded that the accommodative power depends on the distance of convergence while the accommodation of middle-aged subjects is weak in viewing 3D images. Moreover, statistical comparisons revealed that viewing the 3D film on the HMD significantly affected the body sway, despite of a large visual distance.

Key words: Accommodation, Convergence, Visual Fatigue, Visually Induced Motion Sickness (VIMS), Stabilometry

1. Introduction

It is understood that the process by which the eye increases dioptic power to maintain a clear image of an object as it draws near, called “accommodation”, and the simultaneous rotation of both eyes in opposite direction, called “convergence”, are mismatched during single binocular vision, and that this is the main reason for the visual fatigue and the VIMS while viewing stereoscopic video clips. During the test of stereoscopic vision, while accommodation is fixed on the display that shows the 3D image, convergence of left and right eyes crosses at the location of the stereomage. However, according to the findings presented in our previous report (Miyao et al., 1996), these explanations are not correct although our research has not been recognized in the world. This may be because the experimental evidence obtained in our previous studies, where we did not measure accommodation and convergence simultaneously, was not strong enough to convince people. We therefore developed a new device that can measure accommodation and convergence simultaneously.

On the other hand, the human standing posture is maintained by a balance function of body, which is an involuntary physiological adjustment mechanism called “righting reflex” (Okawa et al., 1995). This righting reflex, which is centered in the nucleus ruber, is essential to maintain the standing posture when locomotion is absent. The body’s balance function utilizes sensory signals such as visual, auditory, and vestibular inputs, as well as proprioceptive inputs from the skin, muscles, and joints (Kaga, 1992). The evaluation of this function is indispensable for diagnosing equilibrium disturbances like cerebellar degenerations, basal ganglia disorders, or Parkinson’s disease (Okawa et al., 1996). In general, a stabilometry has been employed for a qualitative and quantitative evaluation of the diagnosing equilibrium function. A projection of a subject’s center of gravity onto a detection stand is measured as an average of the center of pressure (COP) of both feet. The COP is traced for each time step, and the time series of the projections is traced on an x-y plane. By connecting the temporally vicinal points, a stabilogram is created. Several parameters are considered widely in clinical studies to quantify the degree of instability in the standing posture: for instance, the area of sway (A), total locus length (L), and locus length per unit area (L/A). It has been revealed that the last parameter is closely related to the fine variations involved in posture control (Okawa et al., 1995). Thus, the L/A index is regarded as a measure for evaluating the function of proprioceptive control of standing in human beings. However, it is difficult to diagnose disorders of the balance function clinically and to identify the decline in equilibrium function by utilizing the above-mentioned indices and measuring patterns in a stabilogram. Large interindividual differences might make it difficult to understand the results of such a comparison.

Mathematically, the sway in the COP (Fig. 1) is described by a stochastic process (Collins and De Luca, 1993; Emmerek et al., 1993; Newell et al., 1997). We examined the adequacy of using a stochastic differential equation (SDE) and investigated the most adequate equation for our research. G(x), the distribution of the observed point x, is related in the following manner to V(x), the (temporally averaged) potential function, in the SDE, which has been considered to be a mathematical model of sway (Appendix
A): 

\[ V(\vec{x}) = -\frac{1}{2} \ln G(\vec{x}) + const. \]  

(1)

The nonlinear property of SDEs is important (Takada et al., 2001). There are several minimal points of potential. In the vicinity of these points, local stable movement with a high-frequency component can be generated as a numerical solution to the SDE. We can therefore expect a high density of observed COP in this area on the stabilogram.

Watching 3-dimensional (3D) movies can produce certain adverse affects such as asthenopia and motion sickness (Takada et al., 2007). It has been considered that this visually induced motion sickness (VIMS) is caused by the sensory conflict as a disagreement between vergence and visual accommodation while viewing 3D images (Wann et al., 1995). Thus, stereoscopic images have been devised to reduce this disagreement (Yasui et al., 2006; Kakeya, 2007).

The VIMS can be measured by psychological and physiological methods, and the simulator sickness questionnaire (SSQ) is a well-known psychological method for measuring the extent of motion sickness (Kennedy et al., 1993). The SSQ is used in this study to verify the occurrence of VIMS. The parameters of autonomic nervous activity appropriate for the physiological method are the heart rate variability, the blood pressure, the electrogastrography, and the galvanic skin reaction (Holomes and Griffin, 2001; Himi et al., 2004; Yokota et al., 2005). It has been reported that a wide stance (with the midlines of the heels 17 or 30 cm apart) significantly increases the total locus length in the stabilograms of individuals with high SSQ scores, while the length for individuals with low scores is less affected by such a stance (Scibora et al., 2007). We reported that VIMS could
be detected by the total locus length and sparse density, which were used as the analytical indices of stabilograms (Fujikake et al., 2009). This paper explains results of recent studies by the present authors on the problem: why the motion sickness is induced while viewing 3D video clips.

2. Lens Accommodation and Convergence while Viewing 3D Video Clip

In this section, the fixation distances between accommodation and convergence in young and middle-aged subjects are compared, where they viewed 2D and 3D video clips on an LCD (Miyao et al., 2011).

2.1 Eye examination

The subjects in this study were six healthy, young men and women in their twenties and four middle-aged subjects in their forties to fifties. We obtained informed consent from all the subjects and the approval from the Ethical Review Board of the Graduate School of Information Science at Nagoya University.

We placed an LCD monitor facing the subjects at a distance of 1 m from them. We presented either a 2D or a 3D video clip on the monitor, where a spherical object moved forward and backward with a cycle of 10 s (Fig. 2).

The spherical object appearing as a 3D video clip was located at a virtual distance of 1 m from the location of the LCD monitor and moved toward the subjects to a virtual distance of 0.35 m in front of them. We asked the subjects to gaze at the center of the spherical object for 40 s and measured their lens accommodation and convergence distance during that time. The 3D video clip was presented using a liquid-crystal-shutter system and a circular polarizing filter system. The 2D video clip was presented through only a liquid crystal shutter system.

We developed an original machine by combining WAM-5500® and EMR-9® to perform the measurements. WAM-5500 is an auto refractometer (Grand Seiko Co., Ltd.) that can measure accommodative power under natural conditions for the case in which both eyes are open. It can continuously record accommodative focus distance at a rate of 5 Hz. EMR-9 is an eye mark recorder (NAC Image Tech. Inc.) that can measure the convergence distance using the pupillary/corneal reflex method.

We used a liquid crystal shutter system or a circular polarizing filter system combined with the respective binocular vision systems to present 2D and 3D video clips. The experimental environment is shown in Fig. 3. The video clips used in this experiment are trade-marked as Power 3D® (Olympus Visual Communications, Corp.), which is an image creation technique that combines near and far views in a virtual space and has multiple sets of virtual displays whose positions can be adjusted. Power 3D presents a video clip that is similar to a natural image.

2.2 Agreement between lens accommodation and convergence

The measurements showed roughly similar results in each age group. For 3D vision, the results for Subject A (23-year-old male wearing soft contact lenses) and Subject B (45-year-old male, emmetropia) are shown in Figs. 4 and 5, as examples.

In the following, the distances of the convergence and the accommodation were measured by a unit “diopter”, which is a reciprocal of the focus length. When the subject was viewing the 2D video clip, the diopter values for both accommodation and convergence remained almost constant at around 1 diopter (1 m). There is not much quantitative difference in the fixation distances between accommodation and convergence when the subject views either the 2D or 3D video clip.

When Subject A and Subject B viewed the 3D video clips presented, accommodation of the young subject varied between 1 diopter (1 m) and 2.5 diopters (40 cm), whereas convergence varied between 1 diopter (1 m) and 2.7 diopters (37 cm). Accommodation of the middle-aged subject varied between 1 diopter (1 m) and 2 diopter (50 cm), whereas convergence varied between 1 diopter (1 m) and 5 diopters (20 cm). The changes in the respective values were fluctuating synchronously with a cycle of 10 s.

3. Body Sway while Viewing 3D Video Clip

The objective of the present section is to compare the degree of motion sickness induced by viewing a conventional 3D movie on a liquid crystal display (LCD) with that from viewing a 3D movie on a head-mounted display (HMD) which is widely spread in use due to advantages of being individually wearable, small in size, and providing views of a wide visual angle. We measured body sway quantitatively.
Fig. 5. Time course of accommodation, convergence and pupil Diameter. The range of motion in the depth of a 3D sphere was set to 2.88 diopter (1.67–4.55 diopter) while the 3D image moved virtually toward and away from the subject on the LCD.

Fig. 6. Materials used in this experiment; the head mounted display (HMD) and the stabilometer.

during the resting state, exposure to a 3D movie on an LCD, and that on an HMD (Takada et al., 2011).

3.1 Stabilometry

Ten healthy subjects (age, 23.6 ± 2.2 years) voluntarily participated in the study. All of them were Japanese and lived in Nagoya and its surrounding areas. They provided informed consent prior to participation. The following subjects were excluded from the study: subjects working night shifts, those dependent on alcohol, those who consumed alcohol and caffeine-containing beverages after waking up and less than 2 h after meals, those using prescribed drugs and those who may have had any otorhinolaryngologic or neurological disease in the past (except for conductive hearing impairment, which is commonly found in the elderly). In addition, the subjects had to have experienced motion sickness at some time during their lives.

We ensured that the body sway was not affected by environmental conditions. Using an air conditioner, we adjusted the temperature to 25°C in the exercise room, which was kept dark. All the subjects were tested in this room from 10 a.m. to 5 p.m. Three kinds of stimuli were presented on the monitor in random order: (I) a static circle with a diameter of 3 cm (resting state); (II) a conventional 3D movie that showed a sphere approaching and moving away from the subjects, irregularly; and (III) the same motion picture as shown in (II). Stimuli (I) and (II) were presented on an
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Fig. 7. Typical stabilograms (called here “control”) observed when subjects viewed the static circle (a), the conventional 3D movie on the LCD (b), and the same 3D movie on the HMD (c).

LCD monitor (S1911- SABK, NANA0 Co., Ltd.). The distance between the LCD and the subjects was 57 cm. On the other hand, the subjects wore an HMD (iWear AV920; Vuzix Co., Ltd.) during exposure to the movie (III). This wearable display (Fig. 6) is equivalent to a 62-inch screen viewed at a distance of 2.7 m.

The subjects stood without moving on the detection stand of a stabilometer (G5500; Anima Co., Ltd.) in the Romberg posture (Figs. 1 and 6), with their feet together for 1 min before the sway was recorded. Each sway of the COP was then recorded at a sampling frequency of 20 Hz; the subjects were instructed to maintain the Romberg posture for the 60 s. The subjects viewed one of the stimuli, that is, (I), (II), or (III), from the beginning till the end. Before and after the test, they filled out items that were rated with the scale from none, slight, moderate to severe. These subjective questioners called SSQ are composed of a list of 27 symptoms that are commonly experienced as a motion sickness. Four representative scores can be calculated as Disorientation-related subscore (D), Nausea-related subscore (N), Oculomotor Discomfort-related subscore (OD), and Total Score (TS). The disorientation, the nausea, and eyestrain (oculomotor discomfort) are known as representative aspects for the symptoms of the motion sickness. The TS is regarded as a score indicating the overall severity of 3D sickness in this study.

We calculated several indices that are commonly used in the clinical field (Suzuki et al., 1995) for stabilograms, including the “area of sway”, “total locus length”, and “total locus length per unit area”. In addition, new quantification indices, SPD $S_2$, $S_3$ (Appendix A), and total locus length of chain (Takada et al., 2003) were also estimated. We calculated a probability $p$, called a P-value, quantifying the significance of the difference in these sway values. The P-value is compared to significance level set to be 0.05 in this paper.

3.2 Effects of displays on equilibrium function

The SSQ results are shown in Table 1 and include the nausea (N), oculomotor discomfort (OD), and disorientation (D) subscale scores, along with the total score (TS) of the SSQ. No statistical differences were seen in these scores among the stimuli presented to the subjects. However, there

![Graph showing total locus length for Resting State, LCD, and HMD](image)

Fig. 8. Typical results of Nemenyi tests for the following indicators: total locus length (a) and SPD (b) (Takada et al., 2011).
were increases in the scores after exposure to the conventional 3D movies. Although there were large individual differences, sickness symptoms seemed to appear more often with the 3D movies.

Typical stabilograms are shown in Fig. 7. In these figures, the vertical axis shows the anterior and posterior movements of the COP, and the horizontal axis shows the right and left movements of the COP. The sway amplitudes that were observed during exposure to the movies (Figs. 7b and 7c) tended to be larger than those of the control sway (Fig. 7a). Although a high COP density was observed in the stabilograms (Fig. 7a), this density decreased during exposure to the movies (Figs. 7b and 7c).

According to the Friedman test, a non-parametric test, which is used for one-way repeated measures analysis of variance by ranks, the main effects were seen in the indices of the stabilograms, except for the chain ($p < 0.01$, which means strong evidence against the null hypothesis in favor of the alternative). Nemenyi tests, where all classifiers are compared to each other, were employed as a post-hoc procedure after the Friedman test (Fig. 8). Five of the six indices were enhanced significantly by exposure to the 3D movie on the HMD ($p < 0.05$). Except for the total locus length, there was no significant difference between the values of the indices measured during the resting state and exposure to the 3D movie on the LCD ($p < 0.05$).

4. Mathematical Model of the Body Sway

In this section the complexity of the bio-signal or the degree of visible determinism generating those signals are discussed on the basis of our mathematical model including a non-linear analysis.

On the basis of stabilograms given in Sec. 3, a theory has been proposed to obtain a system of stochastic differential equations (SDEs) as a mathematical model of body sway. The anterior-posterior direction $y$ is considered to be independent of the medial-lateral direction $x$ (Goldie et al., 1989). The following set of SDEs on the Euclid space $\mathbb{E}^2 \ni (x, y)$ have been proposed as mathematical models for generating stabilograms (Collins and De Luca, 1993; Emmerrik et al., 1993; Newell et al., 1997; Takada et al., 2001).

$$\begin{align*}
\frac{dx}{dt} &= -\frac{\partial}{\partial x} U_x(x) + w_x(t), \\
\frac{dy}{dt} &= -\frac{\partial}{\partial y} U_y(y) + w_y(t),
\end{align*}$$

where $w_x(t), w_y(t)$ are white noise terms, and $U_x, U_y$ express their temporally averaged potential functions. The stabilograms (Fig. 7) along with Eq. (1) revealed that the functions $U_x$ and $U_y$ have more than one minimal point (Fig. 9), and fluctuations could be observed in the neighborhood of these points (Takada et al., 2001). The variance in the stabilogram depends on the form of the potential function in the SDE; therefore, the SPD is regarded as an index for its measurement.

We put a question whether a coefficient of the random force (see Eq. (A2) in Appendix A and the paragraph below it), could be eliminated from the mathematical model of the body sway. Using our Double-Wayland algorithm (Takada et al., 2005, 2006), we evaluated the degree of visible determinism in the dynamics of the COP sway (for the Wayland algorithm see Appendix B). Representative results of the Double-Wayland algorithm are shown in Takada et al. (2011). The translation error is a statistical index that measures the smoothness of flow in an attractor generating time series (see Appendix B). In addition, randomness can be evaluated by the Double-Wayland algorithm by comparing the translation errors in the temporal differences of the time series (differenced time series) with the results of the Wayland algorithm in each embedding space (Wayland et al., 1993).

If a time series is produced from a chaos process, the translation vectors point to almost the same direction unless the following delay time $\tau$ is too large, since deterministic aspects remain in time development. If we only resample the time series at every delay time $\tau$, components of the delay coordinate (see Appendix B) cannot linearly correlate with each other where $1/\Delta t$ is 20 Hz (Sampling frequency). The authors estimated the auto-correlation func-
tion $\rho(t)$ from the time series data (Matsumoto et al., 2002) and regarded as zero when $\rho(t)$ decreased below $1/e \cong 0.37$ for the first time ($t \geq 0$).

The minimum translation error would be estimated in such an embedding space that has no false intersection along the orbit and best reflects the degree of freedom. Hereby, the optimum embedding dimension to capture the chaos process can be obtained. The differenced time series produced in the stochastic process often reconstitutes an indifferentiable orbit in embedding space. This indicates that the translation error estimated from the differenced time series exceeds the translation error estimated from the time series data. Accordingly, we weighed the translation error estimated from the time series data against the error estimated from the differenced time series in $m$-dimensional phase space.

We calculated translation errors $E_{\text{trans}}$ derived from the time series. The translation errors were also derived from their temporal differences (differenced time series), which are denoted by $E'_{\text{trans}}$. Regardless of whether a subject was exposed to the 3D movie, the $E'_{\text{trans}}$ was approximately 1. Thus, $E_{\text{trans}} > 0.5$ was obtained using the Wayland algorithm (Wayland et al., 1993), which implies that the time series could be generated by a stochastic process in accordance with a previous standard (Matsumoto et al., 2002).

This threshold value 0.5 is half of the translation error resulting from a random walk. The body sway has previously been described by means of the stochastic processes (Collins and De Luca, 1993; Emmerik et al., 1993; Newell et al., 1997; Tåkada et al., 2001), which were shown by the use of the Double-Wayland algorithm. The translation errors estimated by the Wayland algorithm were similar to those obtained from the temporal differences. The exposure to 3D movies would not change the dynamics into a deterministic one. Mechanical variations were not observed in the locomotion of the COP. We assumed that the COP was controlled by a stationary process, and the sway during exposure to the static control image (I) could be compared with that when the subject viewed the conventional 3D movie ((II) or (III)). The indices for the stabilograms might reflect the coefficients in stochastic processes, although no significant difference in translation error was seen in a comparison of the stabilograms measured during exposure to (I) and the conventional 3D movie.

5. The Cause of Motion Sickness Induced by 3D Video Clip

As explained in Sec. 2, we used 2D and 3D images with a virtual stereoscopic view. However, there is a possibility of influences of age and visual functions on stereoscopic recognition. These factors were examined, and it was found that the accommodation and convergence change similarly in young subjects at a constant cycle of 10 seconds, synchronously with to the movement of the 3D image. In the middle-aged subjects, accommodation showed less change than convergence.

Wann et al. (1995) stated that within a virtual reality system, the eyes of a subject must maintain accommodation at the fixed LCD screen, despite the presence of disparity cues that necessitate convergence eye movements to capture the virtual scene. Moreover, Hong and Sheng (2010) stated that the natural coupling of eye accommodation and convergence while viewing a real-world scene is broken when viewing stereoscopic displays.

We simultaneously measured accommodation and convergence for subjects viewing 2D and 3D video clips. Among young subjects, the difference in the eye functions for accommodation and convergence is equally small in the cases of the observation of both 2D and 3D video clips. We do not focus on the displays while viewing 3D images but on the virtual images. However, the stereoscopic images does not look blurred (Miyao et al., 2012). These results deny the hypothesis stated in Sec. 2 that the motion sickness induced by 3D video clips is caused by the sensory conflict as a disagreement between convergence and visual accommodation.

In contrast, the middle-aged subjects showed weak accommodation for 3D video clips. They can view 3D images stereoscopically with weak accommodation power. They are supplemented with a deepened depth of field from pupil (black spot on the eyeball) contraction. Thus, it is concluded that images with left-right parallax can be perceived with contraction of pupil diameter. On the other hand, pupil contraction implies that a decreased amount of light enters the retina. Therefore, middle-aged people might perceive objects as those being darker than younger people do.

The motion sickness induced by 3D video clips might be caused by the sensory conflict as a disagreement between visual and vestibular inputs as follows. We have reported that the VIMS could be detected with the total locus length and sparse density, which were used as analytical indices of stabilograms. In Sec. 3, we showed an analysis of the severity of motion sickness induced by 3D video clips on an LCD compared to that induced by viewing the video clips on an HMD, and discussed on the body sway models. We measured the body sway in a resting state and during exposure to a conventional 3D film on an LCD and an HMD. Friedman tests showed the main effects in the above-mentioned indices for the stabilograms. Multiple comparisons revealed that viewing the 3D film on the HMD significantly affected the body sway, despite a large visual distance. Hence, there

![Fig. 10. The apparent size of displays in this study.](image-url)
are factors of stereoscopic images inducing the motion sickness except for the size of displays.

Regardless of the display on which the 3D movies were presented, multiple comparisons indicate that the total locus length during exposure to the stereoscopic movies is significantly larger than that during the resting state (Fig. 8b). As shown in Figs. 7b and 7c, obvious changes in the form and coefficients of the potential function (1) is working. Structural changes might occur in the time-averaged potential function (1) owing to the exposure to stereoscopic images, which are considered to reflect the sway in the center of gravity. Therefore, the decrease in the gradient of the potential increases the total locus length of the stabilograms during exposure to the stereoscopic movies. The standing posture becomes unstable because of the effects of the stereoscopic movies. Most values of indices during exposure to the 3D movie on the HMD were significantly greater than those in the resting state, although there was no significant difference between the indices of the stabilograms during exposure to the stereoscopic movies. The next step of this study will involve an investigation with subjects that are middle-aged, their accommodation is weak.

This result denies the hypothesis that the motion sickness induced by 3D video clips is caused by the sensory conflict as a disagreement between accommodation and convergence. The motion sickness induced by 3D video clips might be caused by the sensory conflict as a disagreement between visual and vestibular inputs. However, this disagreement should be examined more carefully in future studies.

Appendix A. Stochastic Process and Index

Various stochastic processes are expressed by the following stochastic differential equation (SDE) for a random variable $z(t)$:

$$\frac{dz}{dt} = -f(z) + w(t), \quad (A.1)$$

where $w(t)$ indicates a white noise and $f(x)$ is a non-linear equation. This SDE has already been applied to describe stabilograms (Takada et al., 2001). We have proposed a method to construct a SDE as a mathematical model for these non-Gaussian time series data under the following assumptions:

**Assumption 1** $z(t)$ is one of Markov processes that are determined for $t(> t_0)$ and $z(t_0)$.

**Assumption 2** $z(t)$ is not an anomalous process such as those diverging rapidly in short time.

Based on these assumptions, the stochastic process can be described by the Fokker-Planck equation (FPE). The FPE is rewritten to the following equation for the distribution function of the random variable $g(z|z_0, t)$

$$\frac{\partial g(z|z_0, t)}{\partial t} = \frac{\partial}{\partial z} \left[ f(z) g(z|z_0, t) \right] + \frac{1}{2} \frac{\partial^2}{\partial z^2} g(z|z_0, t) \quad (A.2)$$

by a variable transformation to normalize the 2nd term of the FPE (Goel and Richter, 1978), i.e. what we call FPE-normalization. Herein, the $g(z|z_0, t)$ indicates a conditional probability of process $z(t)$ after the initial condition $z_0$. Equation (A.2) does not have a coefficient corresponding to the diffusivity in the second term because variables are normalized. Calculating the moment of transition probability, this FPE is uniquely corresponding to Eq. (A.1) of the additive type of the SDE. The curved surface $f(z) = 0$ is regarded as an equilibrium space for the SDE in the meaning of the temporal average (a temporal averaged equilibrium space), and the space integral of the function $f(z)$ as a temporal averaged potential function $V(z)$, mathematically. It is generally regarded that the potential function fluctuates along with time.

Deriving the continuous equation from Eq. (A.2), it has been indicated that the term in the left hand side of Eq. (A.2)

$$f(z)g(z|z_0, t) + \frac{1}{2} \frac{\partial}{\partial z} g(z|z_0, t)$$

is regarded as stochastic flow $J$. Setting the left hand side of Eq. (A.2) to be 0, Harken (1975) obtained a stationary solution $g(z)$ to the FPE (A.2) under a natural boundary condition $J(\pm \infty) = 0$. The stationary solution can be regarded as a density distribution through a long observation. Using the temporally averaged potential function $V(z)$, we can write down the relation between the stationary distribution $g(z)$ and the temporally averaged potential function $V(z)$ as

$$V(z) = -\frac{1}{2} \log \frac{g(z)}{C}, \quad (A.3)$$

where $C$ is an integral constant. It is important to analyze forms of the histograms corresponding to the temporal averaged potential functions.

Thus, the stationary solution is regarded as a density distribution with the normalized histogram of the time series data. That is, one can estimate the SDE as a mathematical model for time series data on the basis of Eq. (A.3). Moreover, Takada et al. (2003) indicates that the sparse density depends on the characteristics of the stabilogram and the minimal structure of the temporally averaged potential function.
Appendix B. Wayland Algorithm

The attractor of a chaotic system is constructed by means of embedding the time series data proposed by Takens (1981) in the phase space. The embedding is a process to draw an orbit in a N dimensional phase space (embedding space) by defining a vector whose components at the time t are the values of the data at \( t, t + \Delta t, t + 2\Delta t, \ldots, t + (N-1)\Delta t \), which is denoted by \([x(t)]\) and called the delay coordinates. The parameters N and \( \Delta t \) are referred to as the embedding dimension and the sampling time, respectively.

The Wayland algorithm assumes that the difference vectors \( v(t) = x(t + \tau \Delta t) - x(t) \) in the embedding space characterize the nonlinear variations of the trajectories and estimate the translation error \( E_{\text{trans}} \) in the phase space. The embedding is a process to estimate the translation error \( E_{\text{trans}} \) defined as follows in a m-dimensional embedding space \((m = 1, 2, \ldots, 10)\). From the delay coordinates \([x(t)]\), one can reconstruct a continuous trajectory without false intersections in the embedding space of high dimension. Generally, the dimension of the mathematical model such as dynamical equation systems (DES) can be estimated as a fractal dimension by using the Grassberger-Procaccia algorithm, which is considered to be beneficial for ensuring accuracy. It is possible to estimate the dimension of attractor systems (DES) can be estimated as a fractal dimension by regression (Ruelle, 1989). Therefore, it is difficult to examine whether a stochastic system is suitable for a mathematical model of time series data. It is also known that \( D_2(m) \) estimated from the fractional Brownian motion (Mandelbrot and Van Ness, 1968) saturates in high dimensional embedding space. Moreover, it is not easy to complete this computation because the fractal dimension of the DES is derived from the calculations of all the points in the embedding space. In contrast, visible determinism can be estimated statistically in the case of the Wayland algorithm. The use of statistics shortens the computation time.

A linear correlation between adjacent vectors \( x(t) \) and \( x(t + \tau \Delta t) \) is eliminated by resampling the time series with respect to each embedding delay \( \tau \) (time steps). The procedure of the Wayland algorithm is described as follows.

(i) A series of delay coordinate vectors \( x(t) \) is embedded in each space.

(ii) \( M \) onset periods \( t_0 \) are randomly selected.

(iii) The values of

\[
E_{\text{trans}}(t_0) = \frac{1}{K+1} \sum_{i=0}^{K} \frac{|v(t_i) - \bar{v}|}{|\bar{v}|} \tag{B.1}
\]

are standardized by the average of the difference vectors at \( K + 1 \) points \( \{x(t_i)\}_{i=0}^{K} \).

\[
\bar{v} = \frac{1}{K+1} \sum_{i=0}^{K} v(t_i) \tag{B.2}
\]

is obtained at every onset period, where the \( K \) points nearest to \( x(t_0) \) are selected as \( \{x(t_i)\}_{i=0}^{K} \).

(iv) The median of the \( M \) values of Eq. (B.1) is extracted.

(v) \( Q \) medians are obtained by repeating the above steps. The translation error \( E_{\text{trans}} \) is estimated by the expectation value of these \( Q \) medians. The Double-Wayland algorithm includes the following additional steps.

(vi) Translation errors, \( E'_{\text{trans}} \), are derived from temporal differences in the time series data (differenced time series) \( \{x(t + \tau \Delta t) - x(t)\} \) by the Wayland algorithm outlined above.

(vii) If a differential equation system that included stochastic factors was the generator of the time series, the flow would not be smooth. In such a case, a significantly higher number of translation errors might be estimated in the last step than in step (v).

In this study, we set the conditions of the coefficients \( M \), \( K \), and \( Q \) to be 51, 3, and 10, respectively (Wayland et al., 1993).

References


