A Sense of Non-linearity Originated from Form in Multiple-Folds —Tying a Necktie in Semi-Windsor Knot—

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Daily trouble in tying a necktie to get a good balance of the lengths of two tails of the tie was analyzed with an interest in the form of the tie. A sense of non-linearity often encountered in adjusting to get the good balance was elucidated objectively by experiments and a geometrical modeling of multiple-folds which essentially reproduces the tying process.
Key words: Nonlinearity, Form, Multiple-Folds, Necktie, Semi-Windsor Knot

1. Introduction

At the initial process of tying in Semi-Windsor Knot (Fink and Mao, 1999, 2001), we usually dangle a tie from the neck with its wider tail longer than the narrower tail to get a good balance of the lengths of both tails after the tying. However, we sometimes fail to make it and we come to notice that the difference in the lengths of the two tails at the initial stage (designated as “a”) is not linearly related to the final difference of them (designated as “b”). In this paper, the origin of this sense of nonlinearity was investigated by experiments and theoretical analyses.

2. Experimental

Tying process was repeated and the difference of the length of the two tails after the tying was measured as a function of the difference in the initial stage as shown in Fig. 1. One example of “a”: ① and three examples of “b”: ② ③ ④ are indicated. The data points are approximated with two linear lines with the border at $b = 0$, that is, a good balance position.

3. Modeling by Paper Crafts

Since ties are made of cloths and can be distorted during the tying process, we modeled the tying in Semi-Windsor Knot with a paper stripes shown in Fig. 2(a). Making the development elevation, 5 mountain folds lines inclined with 30 degrees of angle from perpendicular lines accompanied by three mountain fold lines less inclined than above fives as shown in Fig. 2(b). Although the mountain fold lines are separated each other, these spaces are reduced to minimum during the tying process of a real cloth tie. Therefore, without loosing the essential features, we can simplify the model to form 8 mountain fold lines connected in series as shown in Fig. 3(a). The length used for 8 times folds show a clear change depending on the portion of the tie. Figure 3(b) is the plot which corresponds to Fig. 1 and again plotted data can be approximated with two linear lines, showing the phenomena were essentially reproduced by the “paper tie”. In the cloth tie, there is one additional process where the Semi-Windsor Knot is once formed loosely around the neck and tightened to its minimum size by pulling down the narrower tail. Therefore, when the Knot was unbound, the difference of the lengths of the two tails is different from that set before the tying as experimentally shown in Fig. 4.

Figure 4 also shows a nonlinear nature again and the process characteristic to real cloth tie also contribute the nonlinearity.

According to Fig. 3(a), the length of the portion used for 8 times folds depends strongly on the position in the total length of the tie. This is elucidated more clearly in Fig. 5.

According to Fig. 5, the length of the portion used for the 8 times folds changes drastically below the zero difference of both tails before 8 times folds are made. This may be the origin of the sense of nonlinearity which we daily encounter when we tie a necktie.

4. Discussion

Here we consider the “nonlinearity” analyzed above again. Let us here consider the stripe with its width increasing linearly with the position along its longitudinal direction as shown in Fig. 6. Here the upper edge and lower edge of the stripe are expressed as

$$y = ax + b \quad (1)$$

$$y = -ax - b \quad (2)$$

respectively.

For simplicity without loosing the essential points, let us set $b = 0$. Then the length used for n times folds is expressed using the starting point of folding x_1 and the final point of folding x_n as follows,

$$X_n - x_1 = x_1[\{(1 + a)/(1 - a)\}^{n-1} - 1]. \quad (3)$$

Equation (3) tells that the length used for $(n-1)$ times folds is proportional to x_1: the starting point of folding under the fixed n. Equation (3) obeys additivity rule, showing it is a liner system. If we consider the variation of $x_n - x_1$ as a function of n, $x_n - x_1$ shows an exponential increase with

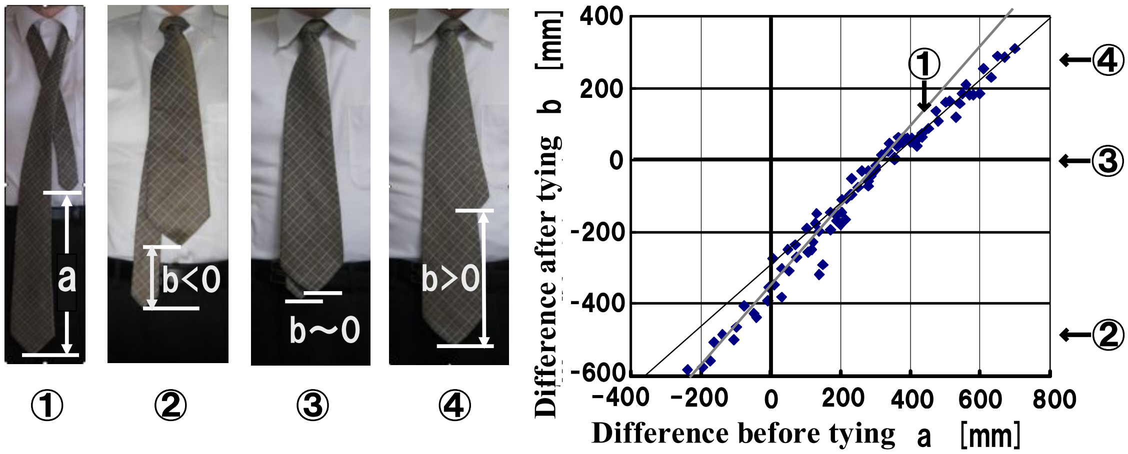

Fig. 1. Measured differences in length of two tails after the tying against those before the tying.

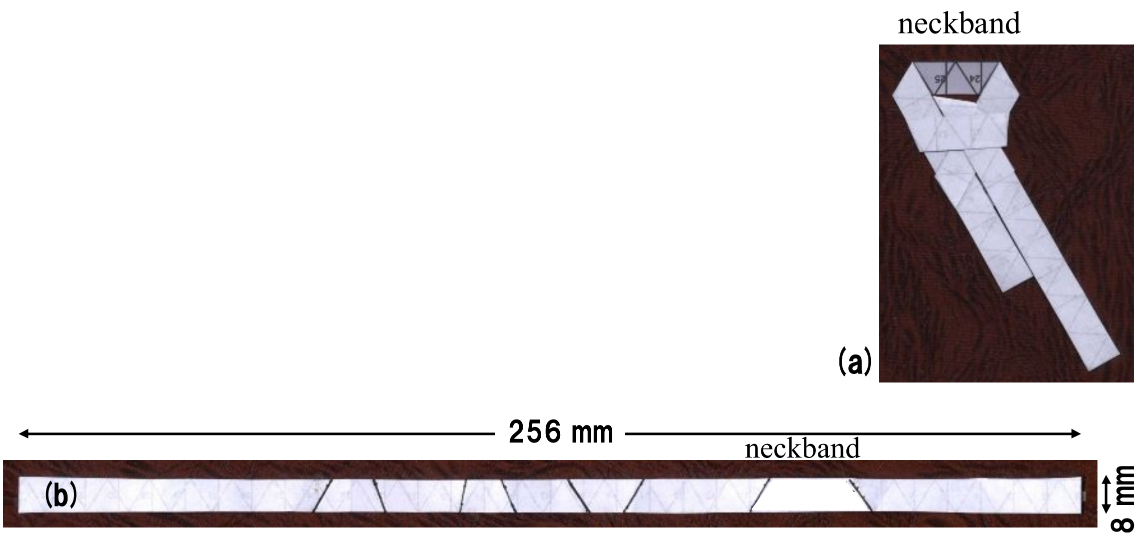

Fig. 2. (a) A paper model of Semi-Windsor Knot and (b) its development elevation. The eight inclined solid lines are lines for mountain fold.

n. This remind us of the Malthus equation representing the growth of population N with time t;

$$dN/dt = \alpha N. \tag{4}$$

Equation (4) obeys under the additivity rule and tells the population grows exponentially with the time.

In Eq. (4), the constant α is called as Malthus coefficient. The simplest modification of the exponential growth of population is done by substituting α with

$$\alpha = \alpha_0 - \alpha_1 N. \tag{5}$$

Then Eq. (4) is transformed as follows;

$$dN/dt = \alpha_0 N - \alpha_1 N^2. \tag{6}$$

This is well-known "logistic equation" giving typically an S-shaped curve as a solution with the saturated value

$$N = \alpha_0/\alpha_1. \tag{7}$$

Obviously, the logistic Eq. (6) does not obey additivity rule and the system is essentially nonlinear.

Following an analogious procedure with Eqs. (4)–(6), we can substitute Eqs. (1)–(3) with

$$a = a_0 + a_1 x \tag{8}$$

which makes the linear Eq. (3) into a nonlinear one.

In this case, the equation representing upper edge and the lower edge corresponding to Eqs. (1) and (2) is expressed as

$$y = a_1 x^2 + a_0 x + b \tag{9}$$
$$y = -a_1 x^2 - a_0 x - b. \tag{10}$$

Figure 7 typically gives a example of this stripes.

Figure 7 can be considered as a simplified model of Fig. 3(a).

5. Conclusion

If we consider the fact that the measured curves in Figs. 1 and 3(b) can be approximated by two straight lines, the necktie is a combined linear system of Fig. 6 with different two value of a, resulting in approaching a nonlinear system

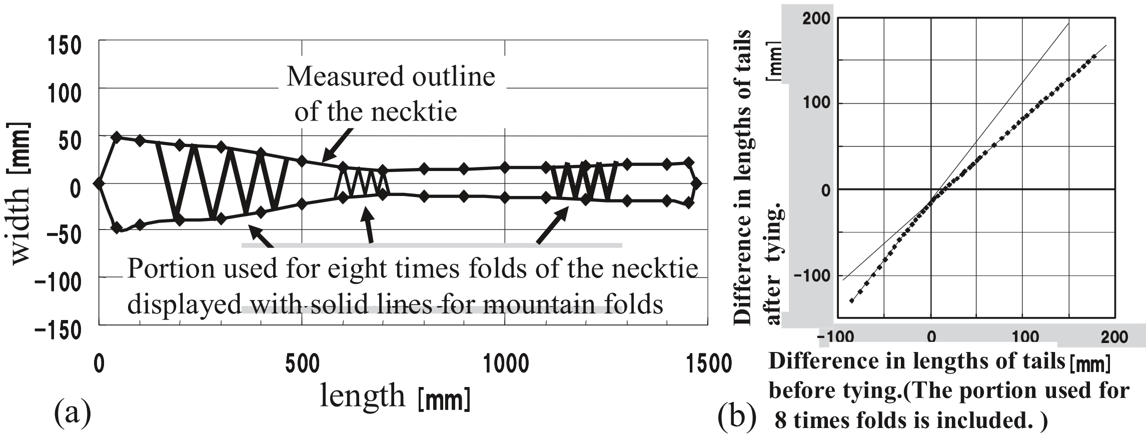

Fig. 3. (a) A drawing figure based on the measured outline of a real necktie showing the portions used for 8 times folds with solid lines indicating mountain folds. (b) A plot of difference in the length of tails after tying against that before that including the portion used for 8 times folds.

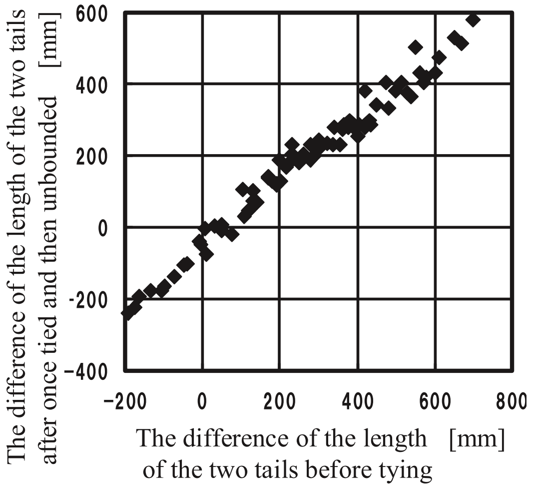

Fig. 4. Relation between the differences of length of the two tails of a tie dangling from the neck before tying and after unbounding of once tied knot.

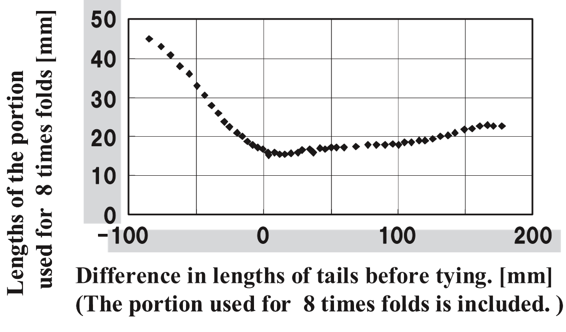

Fig. 5. Dependence of the length of the portion used for 8 times folds on the position to start the folding.

as Fig. 7 giving a sense of nonlinearity in everyday tying procedure.

Addendum

In Fig. 2, the three lines for mountain fold have different inclination angles from those of the other fives, and the reduction of the two lengths of the sevens between the eight lines during the actual tying is limited by the length around the neck. Here, neglecting these factors, the model was simplified without loosing the essential features of the process. Including these factors, it is obviously possible to mark a point on a tie as a function of a length around the neck by extending the analysis described here, which may ease getting a good balance of the lengths of two tails of the tie in tying. However, it is rather out of scope of this paper which tries to point out the sense of "non-linearity" in our daily life in reference to the typical drift of the argument on linearity and non-linearity including the Malthus equation and the logistic equation.

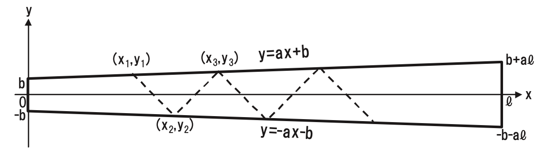

Fig. 6. A stripe with its width start with $2b$ at the left end and increase with the longitudinal length ℓ at the rate of a.

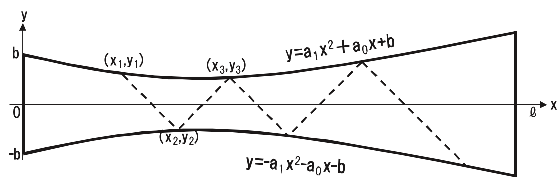

Fig. 7. A stripe with its width start with $2b$ at the left end and change with the longitudinal length ℓ in a quadratic manner.

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