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Extended Pasting Scheme for Kolam Pattern Generation

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Abstract. The theory of "Formal Languages" had been applied successfully to study the aspect of generating patterns of the kolam folk designs. In this paper a new picture generating method by the use of pasting of tiles is presented. The generating model "Pasting Scheme" and its variant "Extended Pasting Scheme" are shown to generate patterns including kolams of kambi and non-kambi types and tessellations as well.

1. Introduction

Kolam is a traditional art of decorating courtyards, temples and prayer rooms in South India drawn mainly by womenfolk. Some women use rice flour to draw a kolam, which is a traditional medium to be used while others use limestone powder. During festivals and weddings, rice flour paste is used instead of the flour. This tradition of decorating with kolams is passed on from generation to generation. Generally dots are drawn first on the floor and then depending on the type of kolam, lines are drawn either connecting the dots or going around the dots and complete as closed curves (Fig. 1). The patterns where the lines are drawn around the dots are called as kambi kolams as they look like wire decoration (kambi means "wire" in Tamil). There are many interesting and complicated designs made up of a single or many closed curves.

These traditional designs also seem to imbibe mathematical properties such as symmetry, permutation etc. The ethno-mathematical view and the corresponding study of kolam patterns have been done extensively by ASCHER (1991) and GERDES (1989). As early as 1974, Rosenfeld advocated a cycle grammar for generation and description of pictures having rotational symmetry. Kambi kolam patterns, fractals provided interesting examples of cycle languages. Kolam designs as examples of two-dimensional picture languages with formally definable syntactic rules, the formal properties of such languages have been studied by SIROMONEY *et al.* (1972). Motivated by the koam designs, SIROMONEY *et al.* (1974) have introduced different types of array grammars generating array languages and also have given specific instructions for drawing certain kinds of kolam patterns using Turtle motions with chain-code interpretation and kolam motions with cycle rewriting rules (SIROMONEY and SIROMONE, 1987; SIROMONEY *et al.*, 1989). Independent of her

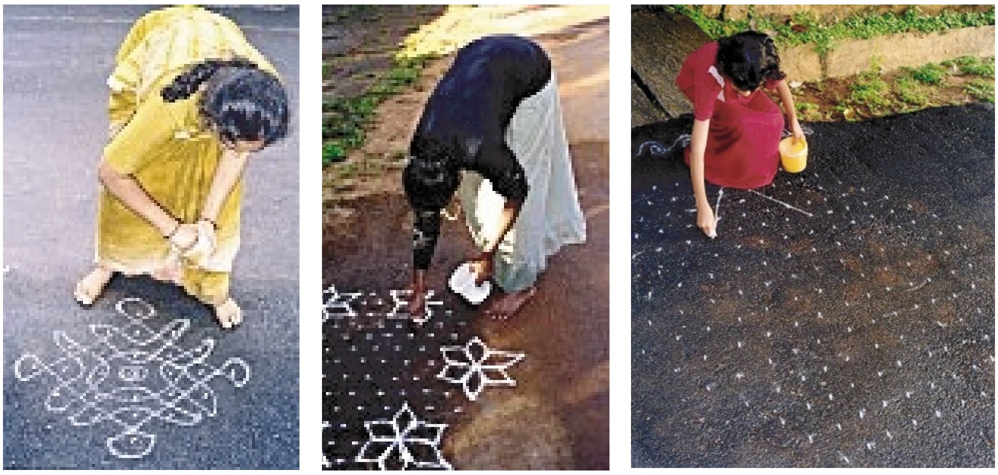


Fig. 1. Women drawing kolam patterns (photo courtesy: Kamat's Potpourri).

studies, computer programs for mechanical or interactive generation of kolam patterns have also been attempted by NAGATA and YANAGISAWA (2004) by the use of other underlying syntax rules and a digital expression for representing Kambi Kolam patterns as well. Recently NAGATA and ROBINSON (2006) have extended the applicability of Kolam designs as tangible pictures for the people with disability.

On the other hand, the art of tiling has been in vogue very early in human civilization. Intricate tiling patterns were used to decorate and cover floors and walls. Motivated by the problems in tiling, NIVAT *et al*. (1995) proposed Puzzle grammars for generating connected arrays of square cells and investigated theoretical questions. A study on tiling patterns by the use of pasting of square tiles at their edges has been done by ROBINSON (2002). KALYANI *et al*. (2006) extended this notion to isosceles triangles and studied certain decidability results. RAGHAVACHARY has used arrangement of diamond shaped tiles by placing them corner to corner to form kolam patterns.

In this paper yet another generating mechanism namely "Pasting Scheme" (PS) and its variant "Extended Pasting Scheme" (EPS) for kolam patterns have been proposed by the use of the notion of pasting of tiles along the specified edges. PS and EPS make use of triangle, square or hexagon tiles, a set of pasting rules for the tiles and a set of constraints. The generative capacity of these models is higher when they include non-kambi kolam patterns too. Finally, irregular polygons as tiles are used to generate patterns, and a tessellation that fills the unbounded space is also attempted.

2. Basic Definitions

Some basic definitions concerning tiles are recollected and necessary notions are illustrated in this section.

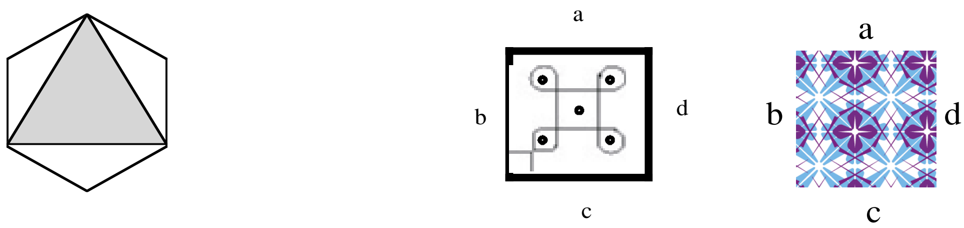


Fig. 2. Decorated hexagonal and square tiles.

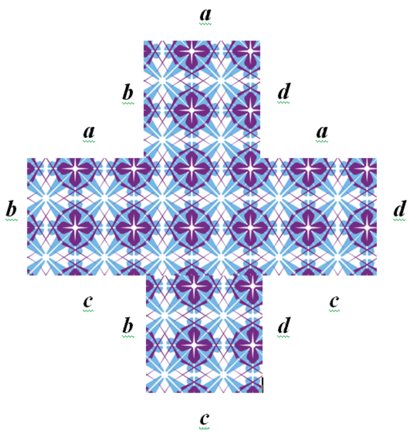


Fig. 3. A tiling pattern with five pasted tiles.

2.1. *Tile*

A tile is a two-dimensional topological disk (region) whose boundary is a single simple closed curve whose ends join up to form a loop without crossing or branches.

2.2. *Tiling*

A plane tiling is a countable family of topological disks which cover the Euclidean plane without gaps or overlaps.

From the definition of tiling, it is clear that the intersection of any finite set of tiles in a tiling has necessarily zero area. Such an intersection will consist of a set of points called vertices and lines called edges. Two tiles are called adjacent if they have an edge in common.

A tile, its vertices and edges may be labeled distinctively. A decorated tile is one whose region is engraved or drawn with any picture/colored pattern/design. A decorated hexagonal tile, a decorated square tile with its edge labels and a decorated square tile with colored patterns are shown in Fig. 2. The boundary of a tile is the sequence of edges taken around the tile in order (generally, anti-clockwise). The boundary edges of second and third tiles of Fig. 2 are the sequence "*abcd*".

2.3. *Pasting rule*

A pasting rule is concerned to a pair of edges of tiles that allows the edges of the corresponding tiles to get glued or attached at those edges. When tiles are attached by pasting rules it results in formation of tiling. The boundary of a tiling is the sequence of edges of the tiles that are exposed to the environment. The pasting rules {(a, c), (b, d)}, which means that the two edges (a, c) are pasted to the two edges (b, d) respectively, are applied to the edges of the tile shown in Fig. 2(c) to obtain a tiling pattern shown in Fig. 3. The boundary of the tiling is "*ababcbcdcdad*"

3. Pasting Scheme for Picture Generation

The formal definitions for Pasting Scheme, Extended Pasting Scheme and derivation of language of tiling patterns by these schemes are given here. Both systems are illustrated with examples to generate kolam patterns and tessellation.

3.1. *Pasting Scheme*

A "Pasting Scheme" (PS) is defined as a 3-tuple, G = (T, P, w_0) where T is finite non-empty set of tiles with labeled edges, P is a finite set of pasting rules and w_0 is the axiom tile or tiling.

By applying pasting rules to the edges of a tile or a tiling t_i, a new tiling t_{i+1} is said to be obtained (or derived) from t_i. It is symbolically denoted as $t_i \Rightarrow t_{i+1}$. Subsequently, the pasting of tiles can be done along the edges of t_{i+1} to derive further tiling patterns. The set of all tiling patterns, L(G) obtained from the given axiom w_0 in a pasting system G is called the language of G. Hence, L(G) = $\{t_i : w_0 \Rightarrow^* t_i\}$ where $\Rightarrow^*$ is the reflexive, transitive closure of $\Rightarrow$. Pasting rules can be applied to all the boundary edges of a tiling t_i simultaneously (parallel application) or sequentially to one edge of t_i at a time. The language of G derived by parallel application of pasting rules is denoted as $L_P(G)$, whereas the language obtained by sequential derivation is denoted as $L_S(G)$. It is obvious that $L_P(G) \subset L_S(G)$.

3.2. *Extended Pasting Scheme*

An "Extended Pasting Scheme" (EPS) is defined as a 4-tuple, H = (T, P, w_0, Δ) where T is finite non-empty set of tiles with labeled edges, P is a finite set of pasting rules, w_0 is the axiom tile or tiling and Δ is a finite set of constraints on the edge labels of T. L(H) is the set of tiling patterns obtained from the axiom w_0 subject to the constraints given in Δ. In other words, the L(H) has all the tiling patterns derived from the axiom which satisfy the conditions of Δ. The application of pasting rules can be either parallel or sequential and hence $L_P(H)$, $L_S(H)$ are obtained accordingly. It can be seen that $L_P(G) = L_P(H)$ and $L_S(G) = L_S(H)$ if $\Delta = \Phi$.

Example 1.

Consider the PS, G_1 = (T, {(14, 22), (14, 32), (24, 32)}, w_0) where T is set of three

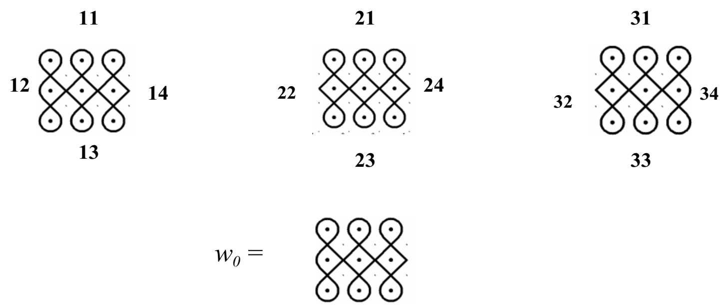


Fig. 4. The set of tiles and the axiom for G_1.

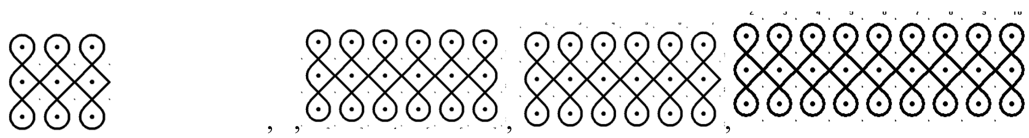


Fig. 5. A sequence of Border kambi kolam patterns.

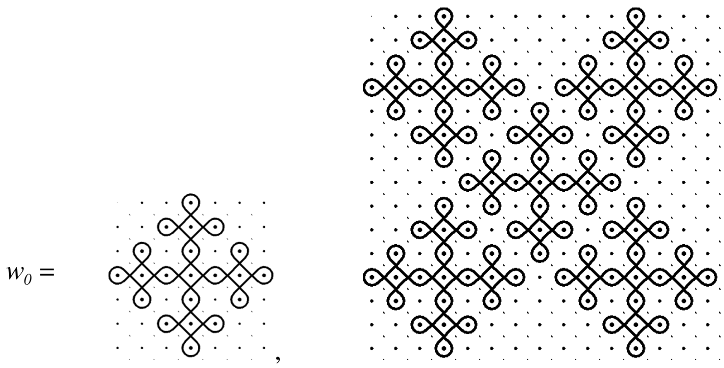
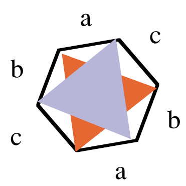


Fig. 6. The first two members by EPS H_1.

decorated square tiles and w_0 is the axiom as shown in Fig. 4. The first four members of the tiling patterns (Border kambi kolam) generated by sequential derivation from the axiom tile is shown in Fig. 5. The patterns engraved on the tiles are highlighted in the derivation of new patterns and not the bounding edges of the pasted tiles. It can be noted that the first and third members in the sequence do not have the smooth curves in the boundary. The PS generates the set of patterns as defined by the pasting rules and it has no control mechanism to generate certain desired patterns. The Extended Pasting System however, has a control

$w_0 =$ 

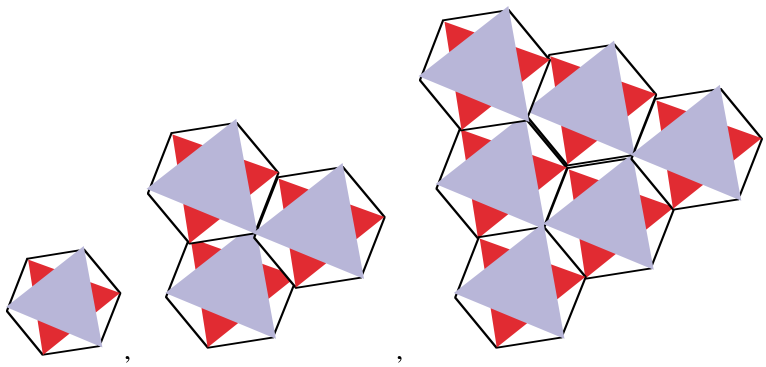


Fig. 7. The first three members by EPS H_2.

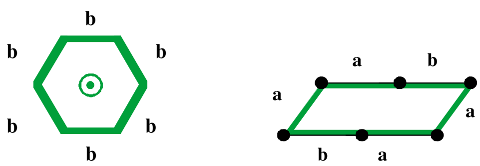


Fig. 8. Hexagon and parallelogram tiles.

to choose certain patterns based on the constraints. In this paper the constraints are used on the boundary labels of the tiling patterns.

Example 2.

Consider the EPS, $H_1 = (T_1, \{(a, d), (b, c)\}, w_0, \Delta_1)$ where T_1 is the square tile as shown in Fig. 2 and $\Delta_1 = \{a, b\}^+$ on boundary of tiling patterns (i.e. the boundary labels of each tiling pattern should be made of a's and b's only). The kambi kolam patterns are generated in parallel and the first two members (including the given axiom) of the sequence are shown in Fig. 6.

Example 3.

Consider the EPS $H_2 = (T_2, \{(a, a), (b, b), (c, c)\}, w_0, \Delta_2)$ where T_2 is the hexagonal

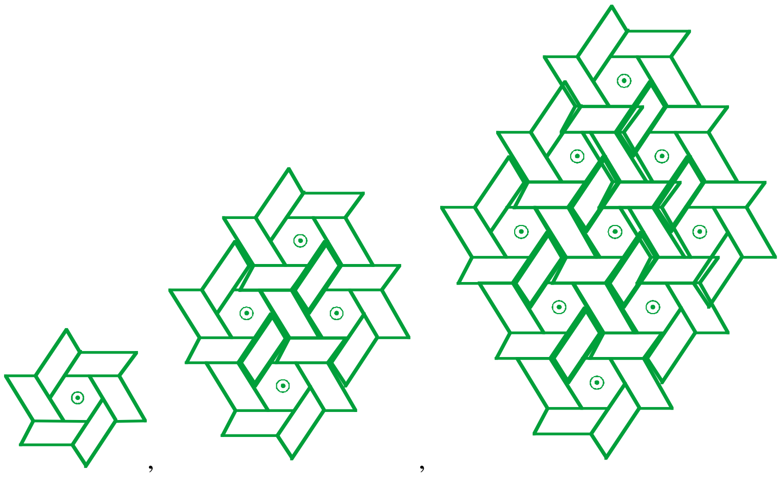


Fig. 9. *Mango leaves* kolam patterns.

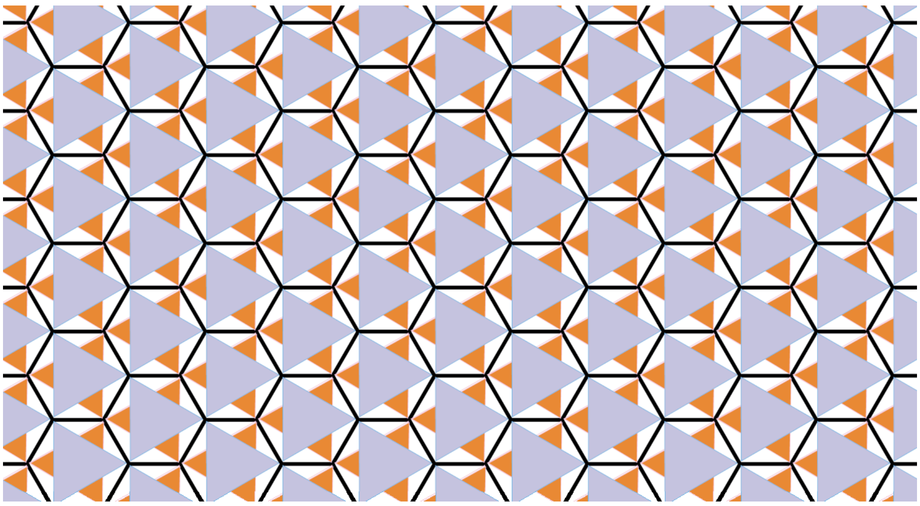


Fig. 10. A tessellation pattern.

colored tile (as well as the axiom w_0) and $\Delta_2 = \{(ab)^n(ca)^n(bc)^n/n \geq 1\}$. The first three members generated sequentially as per the constraint are shown in Fig. 7.

Example 4.

In this example we use irregular polygon tiles to generate a language of kolam patterns (known as "*Mango leaves*"). The EPS $H_3 = (T_3, \{(a, a), (b, b)\}, w_0, \Delta_3)$ where T_3 consists

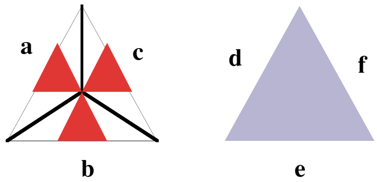


Fig. 11. The triangular axiom tiles.

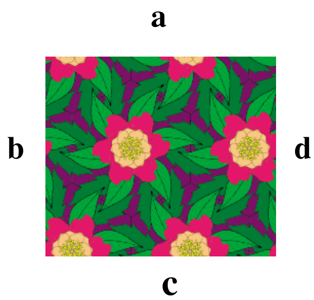


Fig. 12. A decorated rectangular tile.

of a regular hexagon (axiom) and a parallelogram with six labeled edges as shown in Fig. 8.

When $\Delta_3 = \{(ba^2(b^2a^2)^n)^6/n \geq 0\}$, a sequence of patterns with three axes of symmetry is generated. The *Mango leaves* kolam patterns which have single axis symmetry are obtained if $\Delta_3 = \{(ba^2b)^n a^2ba^2(b^2a^2)^{n-1}(ba^2b)^n a^2ba^2(b^2a^2)^{n-1}/n \geq 1\}$. The first three members generated sequentially are shown in Fig. 9.

Example 5.

The derivation of the patterns with no bounding constraints may fill the regions in the Euclidean plane in the limiting case. The tessellation patterns can be generated thus with unlimited boundary constraint (denoted as ∞). For the EPS H_2 considered in Example 3, with $\Delta_3 = \infty$ the (unbounded) tessellation pattern is generated in the limiting case. A portion of the tessellation is shown in Fig. 10.

The tessellation can be generated with two triangular tiles instead of the hexagonal tile by the rules $\{(a, d), (b, e), (c, f)\}$ and $\Delta_3 = \infty$ with any of the two tiles (Fig. 11) as axiom.

Example 6.

Consider the EPS $H_4 = (T_4, \{(a, c), (b, d)\}, w_0, \Delta_4)$ where the axiom tile is a colored rectangle tile as shown in Fig. 12 and $\Delta_4 = \{a^n b^m c^n d^m/n,\ m \geq 1\}$. The language thus generated has the set of all rectangles of area *mn* square units. One member of the language is shown in Fig. 13.

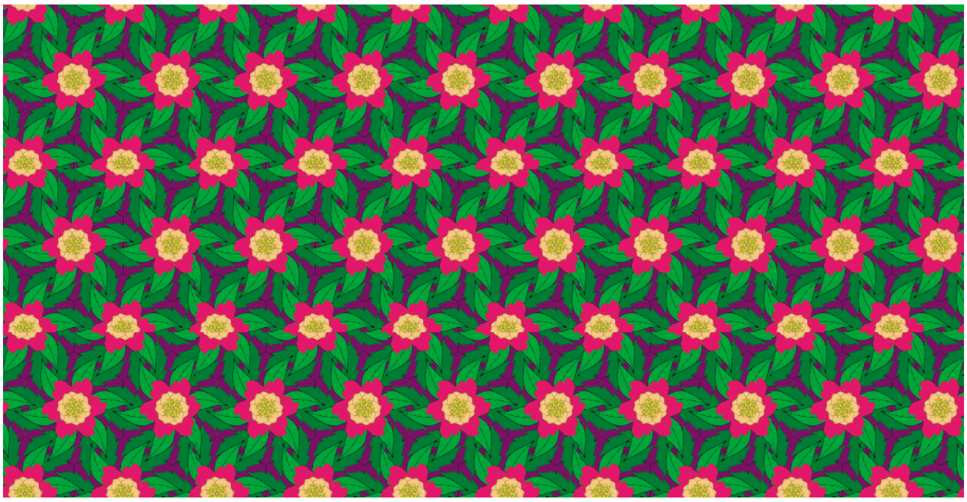


Fig. 13. A tessellation pattern from EPS H_4.

4. Conclusion

A new type of generating system has been introduced in this paper to generate patterns including tessellations, and the suitability of generating kambi and non-kambi kolam pictures has been highlighted. The picture language theoretic study with deterministic pasting rules and decidability results and extension of this notion to pasting of cuboids to form patterns on non-planar surfaces are worth pursuing.

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