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Computer Simulation on the Gumowski-Mira Transformation

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Abstract. We perform a computer simulation on the Gumowski-Mira transformation (hereafter abbreviated as GM tranformation) and present a variety of 2-dimensional patterns obtained from the GM transformation. Among these patterns, there are images which resemble very much “living marine creatures”.

1. Definition of the GM Transformation

One encounters various nonlinear phenomena in nature. Computer simulation provides a powerful tool in understanding such nonlinear phenomena (KINZEL and REENTS, 1998). In this study, we perform a computer simulation on the GM transformation (GUMOWSKI and MIRA, 1980) and present a variety of 2-dimensional images obtained from the GM transformation. Among these images, there are patterns which resemble very much (cross sections of) “living marine creatures”.

The GM transformation is the 2-dimensional discrete dynamic system, which is expressed by the following recurrent formula:

$$x_{n+1} = y_n + a(1 - 0.05y_n^2)y_n + f(x_n),$$

$$y_{n+1} = -x_n + f(x_{n+1}),$$

$$f(x) = \mu x + \frac{2(1-\mu)x^2}{1+x^2}.$$

1.1. *The GM patterns with* a = 0.008

In the following, we set a = 0.008. Changing the value of μ, one obtains very diversified attractors in the x-y plane from the above GM transformation.

The GM patterns, which are obtained by changing μ from μ = – 1.0 until μ = 0.9 with

step $\Delta\mu = 0.1$, are shown in order in Fig. 1.
The more detailed dependence on μ can be seen in Fig. 2, which are obtained by changing μ with step $\Delta\mu = 0.01$ in the range of $0.2 \le \mu \le 0.29$.
Still more detailed dependence on μ can be seen in Fig. 3, which are obtained by changing μ with step $\Delta\mu = 0.0001$ in the range of $0.29 \le \mu \le 0.2909$.
Figures 4 and 5 represent the GM patterns, which are obtained with $\Delta\mu = 0.00001$ in the

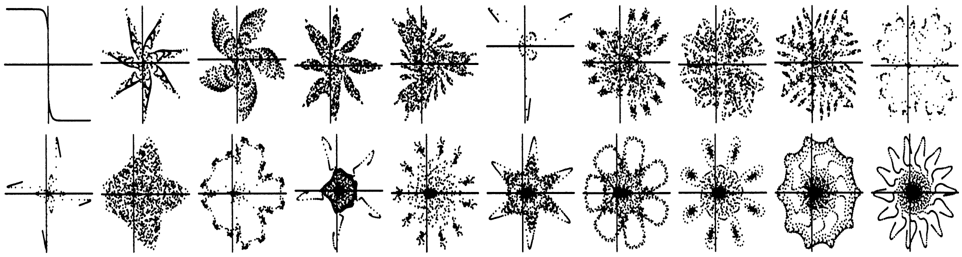

Fig.1. The 2-dimensional patterns obtained from the GM transformation with $a = 0.008$, $\mu = -1 \sim 0.9$, $\Delta\mu$ (step size) = 0.1 and $x_1 = y_1 = 0.1$.

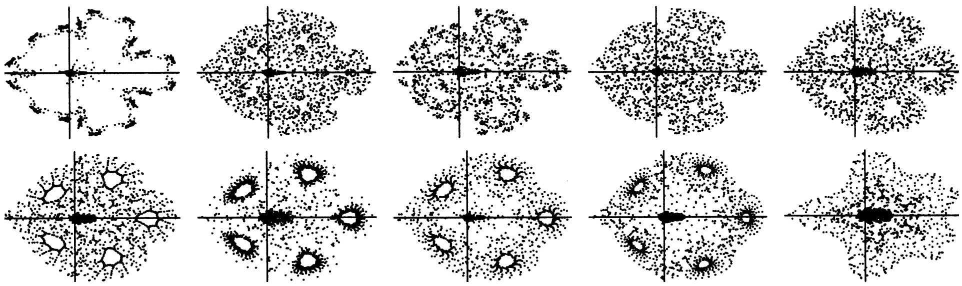

Fig. 2. The GM pattern with $a = 0.008$, $0.2 \le \mu \le 0.29$ and $\Delta\mu = 0.01$.

Fig. 3. The GM pattern with $a = 0.008$, $0.29 \le \mu \le 0.2909$ and $\Delta\mu = 0.0001$.

range of $0.29 \leq \mu \leq 0.29009$, and with $\Delta\mu = 0.0000001$ in the range of $0.29 \leq \mu \leq 0.2900009$, respectively.

From the pictures presented above, one can see that even a small change of μ causes rather drastic changes in the 2-dimensional GM patterns. Resulting patterns depend very sensitively on the value of μ.

Figure 6 represents the attractors obtained with the other values of μ ($-0.8 \leq \mu \leq -0.71$).

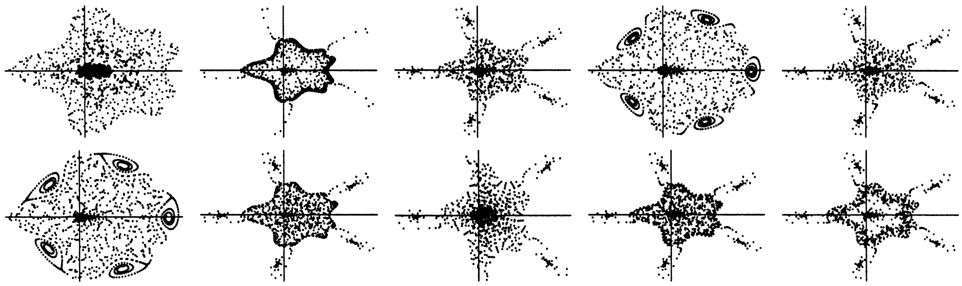

Fig. 4. The GM pattern with $a = 0.008$, $0.29 \leq \mu \leq 0.29009$ and $\Delta\mu = 0.00001$.

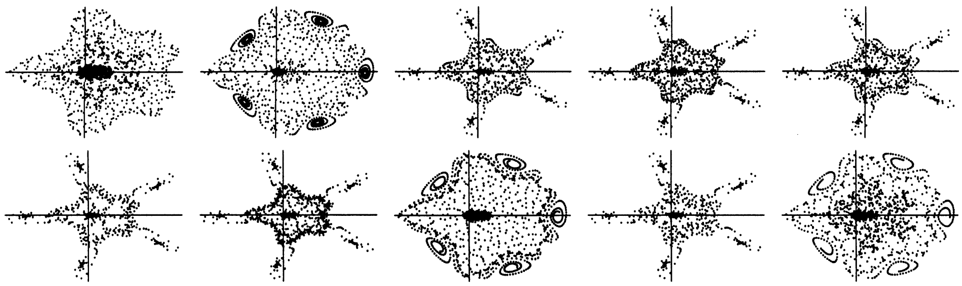

Fig. 5. The GM pattern with $a = 0.008$, $0.29 \leq \mu \leq 0.2900009$ and $\Delta\mu = 0.0000001$.

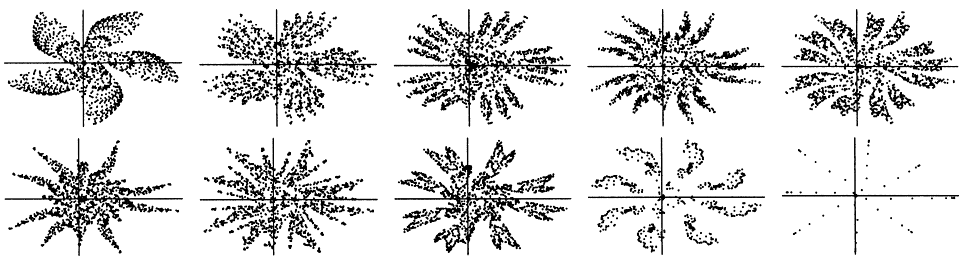

Fig. 6. The GM pattern with $a = 0.008$, $-0.8 \leq \mu \leq -0.71$ and $\Delta\mu = 0.01$.

1. 2. The GM Patterns with a = 0

Next we set $a = 0$. In this case we examine how the attractors look like by changing the value of μ. Figures 7–13 represent the GM patterns obtained with several values of μ. Figures 8–12 resemble a slice of tomato, wings of butterfly, a black butterfly, the cut end of an orange, a plankton, respectively.

2. A Few Remarks on the GM Transformation

2.1. Possible realization of the GM Patterns in nature

It is interesting to notice that these patterns remind us some kind of "living marine creatures" like a jellyfish, a starfish, or a plankton. It is widely recognized that in nature fractal geometry plays a crucial role in forming various self-similar objects like a fern leaf, a ramification of a tree, coastline and so forth. Analogously one is tempted to assume that the shapes of "living marine creatures" appear to be a realization of the GM transformation. From such considerations, one can argue that the mathematics of the complexity systems such as chaos and fractal underlies the living world.

Fig. 7.

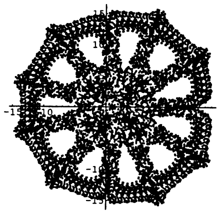

Fig. 8.

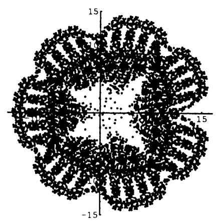

Fig. 9.

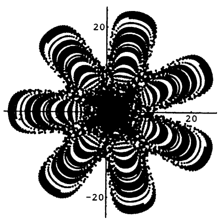

Fig. 10.

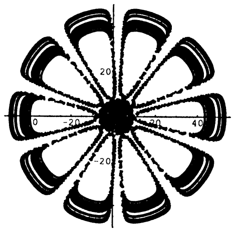

Fig. 11.

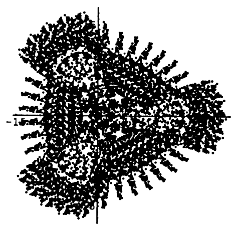

Fig. 12.

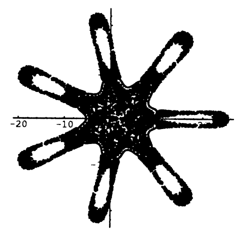

Fig. 13.

Fig. 7. The GM pattern with $a = 0.008$, $\mu =- 0.7$, $x_1 = 0$ and $y_1 = 0.5$.
Fig. 8. The GM pattern with $a = 0$, $\mu =- 0.15$, $x_1 = 0$ and $y_1 = 0.5$.
Fig. 9. The GM pattern with $a = 0$, $\mu = - 0.2$, $x_1 = 0.5$ and $y_1 = 0$.
Fig. 10. The GM pattern with $a = 0$, $\mu = - 0.22$ and $x_1 = y_1 = 0.5$.
Fig. 11. The GM pattern with $a = 0$, $\mu = - 0.31$, $x_1 = 0$ and $y_1 = 0.5$.
Fig. 12. The GM pattern with $a = 0$, $\mu = - 0.55$, $x_1 = 0$ and $y_1 = 0.5$.
Fig. 13. The GM pattern with $a = 0$, $\mu = - 0.23$ and $x_1 = y_1 = 0.5$.

2.2. Bifurcation chart

Figure 14 represents the bifurcation chart, which is obtained in the vicinity of μ = 1.0004. This picture is similar to the one of the logistic map (KINZEL and REENTS, 1998).

2.3. 3-Dimensional image in the (x, y, μ) space

Next, we show a 3-dimensional image of the GM transformation in the (x, y, μ) space by superimposing the 2-dimensional patterns drawn by changing μ with step size $\Delta\mu = 0.04$ (Fig. 15). The various patterns drawn in the 2-dimensional (x, y) space with fixed μ, which themselves represent the cross-sections to the 3-dimensional image, appear to possess some kind of symmetries which are not easy to imagine from the 3-dimensional image.

3. Concluding Remark

As one can easily contrive transformations, which resemble the GM transformation, the study on the other types of transformations which might realize various shapes of the living world should be worthy. We emphasize that such study should help us to get deeper insight into the mathematics hidden in the living world (STEWART, 1998).

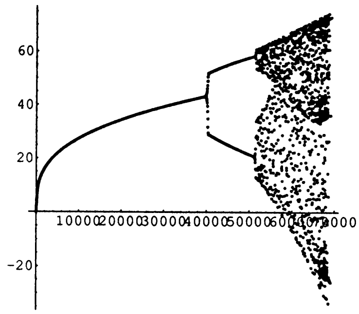

Fig. 14. The GM pattern with $a = 0$, $\mu = 1.0004$, $x_1 = 0.1$ and $y_1 = 0$.

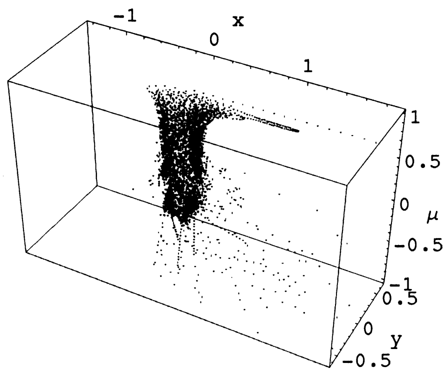

Fig. 15. The 3-dimensional image of the GM transformation in the (x, y, μ) space with $a = 0.008$ and $x_1 = y_1 = 0.1$.

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