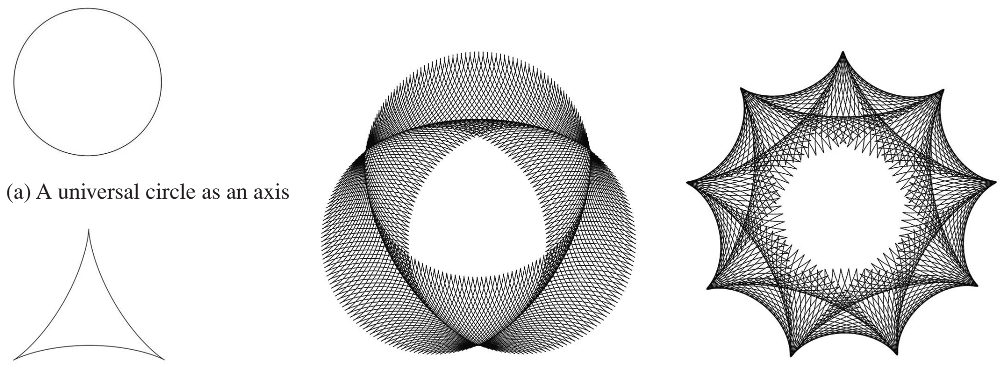
Table 3. Universal circle used as axes and hypocycloid used as motifs ($r_1 = 10, a_2 = 4.5, b_2 = 1.5, c_1 = 1, f_1 : 0.0 \sim 324.0\pi, f_2 : -0.0 \sim 324.0\pi$).

| | $h$ | $d_1$ | $s_1$ | $j$ | $d_2$ | $s_2$ |
|---|---|---|---|---|---|---|
| (c) | 20 | 0.5 | $0.013\pi$ | 40 | — | — |
| (d) | 20 | 0.5 | $0.013\pi$ | 40 | 1 | $0.025\pi$ |



(a) A universal circle as an axis

(b) A hypocycloid as a motif

(c) A Type A geometric pattern

(d) A Type B geometric pattern

Fig. 9. Geometric patterns using combinations of universal circle used as axes and hypocycloid used as motifs.

Figure 9c shows a Type A geometric pattern generated from Formula 20. Figure 9d shows a Type B geometric pattern generated from Formula 22.

### 3.2 Setting for generating geometric patterns using combinations of curves as axes and curves as motifs

The second step is to generate geometric patterns using selected universal cubic curves as the axes and selected quadratic curves as the motifs.

*3.2.1 Curves used as axes*

In the method described in this paper, the curves selected for use as motifs are ellipses, epicycloids, and hypocycloids (which are quadratic curves). These curves are transformed into cubic curves and used as the axes to generate geometric patterns. Each of these transformed cubic curves is named according to the quadratic curve it was created from. For example, a cubic curve transformed from an epicycloid is referred to as a universal epicycloid. This paper treats these curves as the equivalent of knots in topology. Three types of knots were selected unknots. The relationship between a curve transformed into a cubic curve for use as the axes and its knot is as follows:

a. Universal golden ellipse: unknots
b. Universal epicycloid: unknots
c. Universal hypocycloid: unknots

*3.2.2 Curves sed as motifs*

Curves $A_2$ to $C_2$ shown below were selected as the curves used as motifs. In other words, the quadratic curves that are the motifs are the same as the parallel projections of the cubic curves used as the axes.

a. golden ellipse
b. epicycloid
c. hypocycloid

*3.2.3 Combination of curves used as axes and curves used as motifs*

We investigated the generation of geometric patterns by combining the curves used as the axes and the curves used as the motifs. Table 1 shows the combinations ($A_1$ to $C_1$ indicate the figures used as the axes, and $A_2$ to $C_2$ indicate the figures used as the motifs).

Table 4. Curves used as axes and curves used as motifs.

| | $A_2$ | $B_2$ | $C_2$ |
|---|---|---|---|
| $A_1$ | $A_1$-$A_2$ | $A_1$-$B_2$ | $A_1$-$C_2$ |
| $B_1$ | $B_1$-$A_2$ | $B_1$-$B_2$ | $B_1$-$C_2$ |
| $C_1$ | $C_1$-$A_2$ | $C_1$-$B_2$ | $C_1$-$C_2$ |

### 3.3 Generating geometric patterns using combinations of curves used as axes and curves used as motifs

Using the combinations shown in Table 4, we investigated the generation of geometric patterns by combining the curves used as the axes and the curves used as the motifs. Type A geometric patterns were created by restructuring the curve's equation with Formula 8, and Type B geometric patterns by restructuring the curve's equation with Formula 9. Figures 10 to 18 show the generated geometric patterns. For each diagram, Fig. 10(a) is the curve used as the axes, Fig. 10(b) is the curve used as the motifs, Fig. 10(c) is a Type A patterns, and Fig. 10(d) is a Type B patterns.

*3.3.1 Using universal golden ellipses as axes*

*a. Using golden ellipse as motifs ($A_1$-$A_2$)*

Formula 1 is restructured with Formula 8 to express the Type A geometric pattern by Formula 23 ($a_3$: major axis of the golden ellipse as an axis; $b_3$: minor axis of the golden ellipse as an axis; $a_3 = \tau b_3$).

$$\begin{aligned} x_{m6} &= a_3 \sin g_1 + x \\ y_{m6} &= b_3 \cos g_1 + y \\ z_{m6} &= c_3 \sin g_1. \end{aligned} \tag{23}$$

Formula 4 is restructured with Formula 9 to express the