

Table 1. Number sequences for various divergence angles.

Divergence angle	Sequences
137.51°	$F, G(1, 2) : 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, \dots$
99.50°	$L, G(1, 3) : 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, \dots$
77.96°	$G(1, 4) : 1, 4, 5, 9, 14, 23, 37, 60, 97, 157, 254, 411, 665, 1076, \dots$
64.08°	$G(1, 5) : 1, 5, 6, 11, 17, 28, 45, 73, 118, 191, 309, 500, 809, 1309, \dots$
54.40°	$G(1, 6) : 1, 6, 7, 13, 20, 33, 53, 86, 139, 225, 364, 589, 953, 1542, \dots$
47.25°	$G(1, 7) : 1, 7, 8, 15, 23, 38, 61, 99, 160, 259, 419, 678, 1097, 1775, \dots$
151.14°	$G(2, 5) : 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, 343, 555, 898, 1453, \dots$
158.15°	$G(2, 7) : 2, 7, 9, 16, 25, 41, 66, 107, 173, 280, 453, 733, 1186, 1919, \dots$

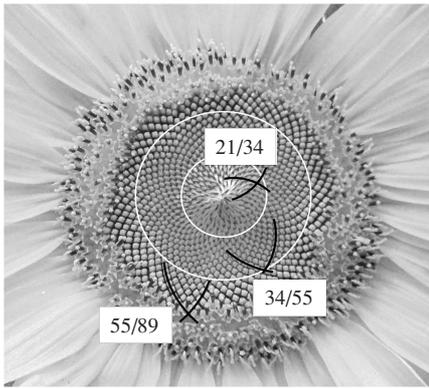


Fig. 1. The seed pattern of a sunflower head displaying Fibonacci numbers. The black and white lines are counting guides marked on the original sunflower.

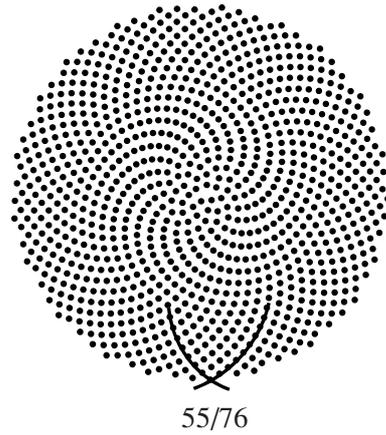


Fig. 2. Simulated point pattern of a sunflower model with a divergence angle of 137.4° and $n = 1000$.

mathematical spiral model to approximate the complex arrangements of the florets in the sunflower head. However, Vogel's spirals lack both translational and orientational symmetry in real space. Accordingly, the Fourier space of Vogel's spirals does not exhibit well-defined Fourier peaks but shows diffuse circular rings, similar to the electron diffraction patterns observed in amorphous solids and liquids (Trevino *et al.*, 2008). This suggests that point distances in a short range are required to analyze spirals. Liew *et al.* (2011) applied the Fourier-Bessel transform to understand the structural complexity of the golden angle spiral. Pennybacker *et al.* (2015) clarified the relationship between parastichy numbers and Fourier decompositions in phyllotactic patterns. We reported a practical method to obtain parastichy numbers using a discrete Fourier transform focusing on the circular symmetry in Fourier space. Fourier transforms are widely used to grasp the characteristics of periodic and aperiodic patterns in natural phenomena and to analyze crystal structures via X-ray diffraction (Authier, 2001; Kikuta, 2011). We tested our method's applicability with sunflower and pineapple models. We applied the discrete Fourier transform to the simulated point patterns of the models. Parastichy numbers for point patterns with various divergence angles were examined in detail.

2. Simulation Models

2.1 Sunflower model

For a sunflower model, the point positions can be determined by the following equation in polar coordinates (r, θ)

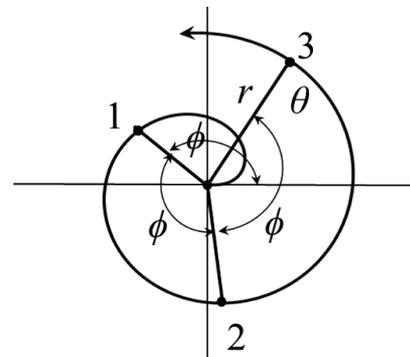


Fig. 3. Spiral trajectory of a sunflower model.

(Fig. 3):

$$(r, \theta) = (n^p, n\phi). \quad (2)$$

Here, n is an integer, p is a constant scaling factor, and ϕ is the divergence angle. When $n = 1000$, $p = 0.5$, and $\phi = \phi_\tau$, the points form the spiral shape shown in Fig. 4. Dominant parastichy numbers can be visually counted toward the outer rim as 21/34, 34/55, and 55/89, which correspond to Fibonacci numbers. The parastichy numbers vary depending on the values of n , p , and ϕ , as we will see later.

2.2 Pineapple model

A pineapple model is expressed by points on the surface of a cylinder, as shown in Fig. 5. The height l and the argu-