various occasions such as the study of disintegration of the invariant curves (Greene, 1979), structure of the intersection of the stable and unstable manifolds of the saddle fixed point (Yamaguchi and Tanikawa, 2001), and construction of the resonance regions (Yamaguchi and Tanikawa, 2009).

In the present paper, we take $f(x) = f_L(x) = (a/2)(1 - |2x-1|)$ for $a \geq 0$. This is called the Lozi map (Lozi, 1978). The map is a piecewise linear version of the connecting map with $f(x) = f_H(x)$. For $a > 0$, we have $f_L(0) = f_L(1) = f_H(0) = f_H(1) = 0$. We also have $f_L'(0) = f_H'(0) = a$ and $f_L'(1) = f_H'(1) = -a$, where the prime denotes the differentiation with respect to the argument, and $f'(0)$, say, is the slope of function $f(x)$ at $x = 0$. The connecting maps and Lozi maps have fixed points $P = (0, 0)$ and $Q = (1, 0)$ for $a > 0$.

The fixed point P is a saddle with eigenvalues $\lambda_\pm$ where $0 < \lambda_- < 1 < \lambda_+$. For $0 < a < 4$, Q is an elliptic fixed point with complex eigenvalues. At $a = 4$, Q undergoes period doubling bifurcation. At $a > 4$, Q is a saddle with reflection with eigenvalues $\lambda_- < -1 < \lambda_+ < 0$.

The Smale horseshoe exists at $a \geq 5.176605\ldots$ (Yamaguchi and Tanikawa, 2009) for the connecting maps, while at $a \geq a_c^{\mathrm{SH}} = 4.229981\ldots$ for the Lozi map. In the Lozi map, the mapping function $f_L(x)$ has a break point at $x = 1/2$. As a result, the stable manifold W_s and the unstable manifold W_u of P have the break points (see Fig. 1). Using the break point, the critical value a_c^{SH} is determined analytically (see Appendix A).

The properties of the horseshoe are discussed in Guckenheimer and Holmes (1983), Gilmore and Lefranc (2002), and Yamaguchi and Tanikawa (2016).

Section 2 is for preparations. We summarize the bifurcations used in this paper and define the dominant axis for T^q for $q \geq 1$. In Sec. 3, we study the bifurcations in the Lozi map. It is shown that the dominant axis theorem does not hold for the Lozi map. In Sec. 4, a new theorem is obtained. In Sec. 5, we give concluding remarks.

2. Mathematical Tools

2.1 Bifurcations

We explain several known terms used in this paper. If the eigenvalues of the linearized matrix are complex, we call the corresponding periodic orbit "the elliptic periodic orbit with complex eigenvalues". In the following argument the cases with $\lambda = \pm 1$ are treated as those with complex eigenvalues for convenience.

If the eigenvalues satisfy relations $\lambda_- < -1 < \lambda_+ < 0$, we call the corresponding periodic orbit "the saddle periodic orbit with reflection". The elliptic periodic orbit with complex eigenvalues and the saddle periodic orbit with reflection will together be called "elliptic" in the present report. If the eigenvalues satisfy relations $0 < \lambda_- < 1 < \lambda_+$, we call this periodic orbit a "saddle" as usual.

In this paper, we use the three bifurcations named rotation bifurcation, period doubling bifurcation and equiperiod bifurcation. In the following, we summarize them.

(i) **Rotation bifurcation**. If the average rotation rate, i.e., rotation number, around elliptic fixed point Q becomes an irreducible fraction p/q satisfying the conditions $0 < p/q < 1/2$, a pair of elliptic and saddle periodic orbits are born. We call this the rotation bifurcation of Q. Bifurcation parameter value is $a = a_c(p/q) = 4\sin^2(\pi p/q)$. We denote the elliptic orbit by p/q-BE, and the saddle orbit by p/q-BS. Here, E in BE stands for "elliptic", S in BS for "saddle", and B in BE and BS for "Birkhoff". The "Birkhoff" comes from mathematician's name who studied the order-preservation property of orbits (Birkhoff, 1966). These are symmetric periodic orbits.

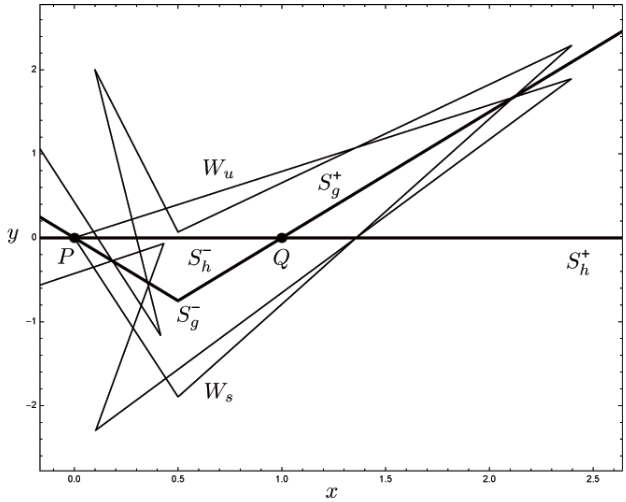

Fig. 1. The branches of symmetry axes S_g^+, S_g^-, S_h^+, and S_h^- are displayed at $a = 3$. The intersection points of symmetry axes are the fixed points P and Q. The stable manifold W_s and the unstable manifold W_u of the saddle fixed point P are also illustrated.

(ii) **Period doubling bifurcation**. The elliptic periodic orbit undergoes period doubling bifurcation if its eigenvalues arrive at $\lambda = -1$ on the complex eigenvalue space. After period doubling bifurcation, the mother orbit becomes a saddle with reflection. A daughter periodic orbit with twice the period appears from the mother point and is elliptic with complex eigenvalues just after the appearance.

(iii) **Equiperiod bifurcation**. The elliptic periodic orbit undergoes equiperiod bifurcation if its eigenvalues arrive at $\lambda = +1$ on the complex eigenvalue space. After the equiperiod bifurcation, the mother orbit becomes a saddle. Two daughter periodic orbits of the same period appear from the mother point and are elliptic with complex eigenvalues just after the appearance.

2.2 Involutions and symmetry axes for T

The Lozi map T is reversible. The set of the fixed points of involution is the symmetry axis. We give the representations of the symmetry axes S_g and S_h.

$$S_g \; : \; y = -f(x)/2, \quad S_h \; : \; y = 0. \tag{3}$$

Here, we also define the branches of symmetry axes.

$$S_g^+ : y = -f(x)/2 \; (x \geq 1), \quad S_g^- : y = -f(x)/2 \; (x < 1). \tag{4}$$

$$S_h^+ \; : \; y = 0 \; (x \geq 1), \quad S_h^- \; : \; y = 0 \; (x < 1). \tag{5}$$

Here, S_g^+ is conventionally called the dominant axis (Dulling *et al.*, 2005). The symmetry axes and the stable and unstable manifolds of P are displayed in Fig. 1.