Science on Form: Proceedings of the First International Symposium for Science on Form,
General Editor: S. Ishizaka, Editors: Y. Kato, R. Takaki, and J. Toriwaki, 585-591.

A Consideration of Geometrical Similarity in the Muscles of Animals of Different Body Dimensions in Relation to Physical Work Capacity

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Keywords: similarity, homomorphism, mass exponent, work capacity

In the forms of animals and their organs, geometrical similarity is commonly observed. However, their metabolic rates are dissimilar, and have a mass exponent of about 2/3. On the other hand, in artificial systems, geometrical similarity is unimportant, and their mass exponent is almost unity. In this paper, the features of natural and artificial systems are discussed comparatively, and the conclusion drawn that nature has an apparent tendency to retain the form of a natural system without variation, while humans, on the other hand, have an apparent tendency to retain the time scale of an artificial system without variation. As a result, the time scale of an animals life has to be adjusted according to its size, and the form of the artificial machine also has to be adjusted accordingly.

INTRODUCTION

In the forms of living organisms, geometrical similarity is commonly observed over a wide range of body size, so that many muscles among animals of different sizes are similar. The heart is a blood pump constructed of muscle fibres, and the heart of one animal is geometrically similar to that of others. There exists a clear relation between heart mass and the body mass of the animal. Statistical results have shown that, among body masses ranging from less than 10 g to more than 100 kg, the slope of the regression line between heart masses and body masses is 0.98 (Stahl 1965). This shows that hearts are not only geometrically similar but also have masses exactly proportional to their body masses. Similarities have also been observed in other organs as well as the whole bodis of animals.

However, such a homomorphic feature of animal organs has not yet been explained as a consequence of a physiological tendency to maximize metabolic efficiency. In order to investigate the origin of the homomorphism observed in animal organs, the mechanical features of muscles were compared with those of artificial machines.

METABOLIC RATE OF MUSCLE

The metabolic rate of an animal can be estimated fairly accurately by measuring the amount of oxygen uptake, and many such studies have been conducted for different animal species. Oxygen is transported only by the blood and thus cardiac output has to be proportional to oxygen uptake. Cardiac output will be proportional to the mechanical power output of the heart as long as the arterial pressure remains within the same range.

The relation between metabolic rate P and body mass M is represented by the mass exponent b and the mass coefficient a as

$$P = a\ M^{b}.$$

Many studies have shown that the mass exponent b is about 0.75 (=3/4), as reviewed by Schmidt-Nielsen (1975) or Kleiber (1975). However, Heusner (1982a,1983) investigated statistical data and concluded that the mass exponent of the metabolic rate was about 0.67 (=2/3) in the same group of animals, and that the mass exponent of 0.75 was a statistical artefact. Although this problem is still controversial, it may be true that the mass exponent of the metabolic rate is around 0.7, which is undoubtedly below unity. The mass exponent of the mechanical power output of the heart will thus also be about 0.7. In the following discussion, therfore, 0.67 or 2/3 will be used for convenience as the mass exponent of the mechanical power output of the heart and other muscular systems.

POWER OUTPUT OF ARTIFICIAL SYSTEMS

There are many artificial systems which, like the muscular system, convert chemical potential into mechanical power. The internal conbustion engine is an example. In Fig. 1, power outputs of reciprocative engines are plotted against their weights. Three types of engines, i.e. 2-stroke-cycle and 4-stroke-cycle gasoline engines and 4-stroke-cycle Diesel engines are shown. The gasoline engine is more powerful than a Diesel engine of the same weight, but is limited in the range of relatively smaller weight. On the contrary, Diesel engines are available over a wider range of weight. The number of cylinders is shown adjacent to each point. The smaller engine has only a few cylinders but the larger engine has many (up to 18) cylinders.

From this chart, two facts can be recognized. The first of these is that the power outputs of Diesel engines are almost proportional to their weight. In other words, the mass exponent of the power output of the Diesel engine is about 1.0. The second is that the number of cylinders increases with the weight. Thus, engines of different sizes are geometrically dissimilar. Engines in practical use are therefore different from muscular systems which are homomorphic with a mass exponent of about 2/3.

COMPARATIVE ANALYSIS OF ENGINES WITH DIFFERENT CONFIGURATIONS

Many different configurations of reciprocative engines can be conceived as long as their economical value is ignored. Among these, three different configurations are assumed as shown in Tab. 1. The fundamental unit is assumed to be a single-cylinder engine which has a practically acceptable performance.

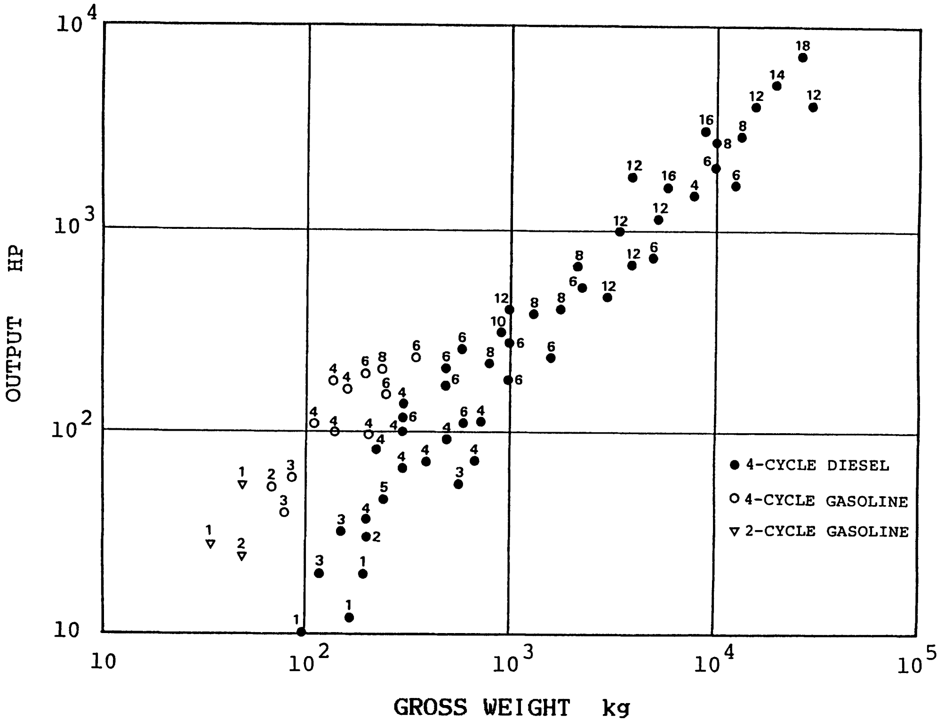
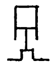
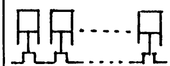
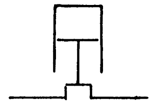


Fig. 1. Engine power output plotted against gross weight. Numbers adjacent to each plot indicate the number of cylinders of the engine. The data was extracted from the Japanese Engine Handbook, 1985, Sankaido, Tokyo.

Next, consider two different types of large engine having a weight eight times greater than the fundamental unit. One has eight cylinders, each of which is the same size as the fundamental unit, the other configuration is a single cylinder but the size is doubled. The expected performances of these engines are listed in the table. The developed force will be proportional to the cross- sectional area of the cylinders. Thus the developed force of an 8-cylinder engine will be 8 times of that of the fundamental one. However, the double-size single cylinder engine will have a developed force only 4 times greater. The work done in one cycle of the double-size engine will be 8 times as much, because the stroke of the double-size engine is twice as much even theough the developed force is 4 times greater. Engine speed can be determined independently from the configuration, however, if the double-size engine operates at the same engine speed, in which case the linear velocity and acceleration of the moving parts are doubled because the stroke is doubled. It is also possible for the engine to operate so that the piston moves at the same velocity as that of the fundamental unit. However, the engine speed is reduced by one

Tab. 1. Mechanical performances of cylinders of different configuration and operating speed.

	SINGLE CYLINDER 	8-CYLINDER 	DOUBLE SIZE SINGLE CYLINDER 	
WEIGHT	1	8	8	
DEVELOPED FORCE	1	8	4	
WORK DONE IN ONE CYCLE	1	8	8	
ENGINE SPEED	1	1	1	0.5
PISTON SPEED	1	1	2	1
POWER OUTPUT	1	8	8	4
POWER OUTPUT PER WEIGHT	1	1	1	0.5
		MECHANICAL SYSTEM		MUSCULAR SYSTEM

half.

The power output of an 8-cylinder engine is 8 times, and the power output per weight is the same as that of the fundamental unit. The power output of the double-size engine is 8 times greater when it operates at the same engine speed, but it becomes 4 times greater at half speed, and in this case, the power output per weight is reduced by one half. In mechanical systems, especially in engines for locomotives, power output per weight is an important feature of merit, and a double-size engine operated at half engine speed is not practically preferable. On the other hand, increased velocity of piston movement will also not be preferable because of various mechanical limitations such as the inertial force arising in the moving parts, the problem of lubrication, or the fluid dynamic requirements of the gas flow. Consequently, the preferable choice may be the many-cylinder configuration operating at the same engine speed as that of the fundamental unit. In practice, large engines have many cylinders as shown in Fig. 1, and realize a power output proportional to their weight, while the number of cylinders is not exactly proportional to the weight or power output.

In contrast to practical engines, muscular systems are homomorphic as in the case of the heart. The performance of a large heart having a weight eight times greater than a small one resembles that of a double-size engine operating at half engine speed. No animal has many heart ventricles, even though a multi-ventricular heart would realize a larger power output per weight than a single ventricular heart. Animals therefore employ an ineffective heart configuration, despite the fact that a more effective design is theoretically possible. Consequently, the mass exponent of the power output of the heart remains at 2/3 instead of 1.0 which is theoretically attainable, and has been realized in the mechanical system.

MASS EXPONENTS OF FUNDAMENTAL PARAMETERS OF THE MUSCULAR AND THE MECHANICAL SYSTEMS

Mass exponents of fundamental parameters expected in idealized muscular and mechanical systems are listed in Tab. 2. Four parameters, i.e. force, power output, speed and acceleration are presented. The mass exponent of force is 2/3 in the muscular system, and 1 in the mechanical system. The mass exponent of power output is also 2/3 in the muscular, and 1 in the mechanical system. The mass exponent of speed is zero in both systems. Acceleration is obtained from the force divided by mass, thus the mass exponent is -1/3 in the muscular, and zero in the mechanical system.

Comparing these relations to the original physical dimension of each variable, as shown in the left-hand column of the table, the mass exponent of L and T can be obtained. In the muscular system, the mass exponent of L is 1/3, and that of T is also 1/3. Substituting these relations into the expressions for the original physical dimensions of these variables, the mass exponents are consistently derived.

The mass exponent of 1/3 in L is consistent with the fact that the muscular systems are geometrically similar. The mass exponent of 1/3 in T, however, means that muscular systems of different sizes have different time scales, or, as Heusner (1982b) stated, "organisms live in their own time depending on their size".

Tab. 2. Mass exponents of fundamental parameters in muscular and mechanical systems, with the conversion of length L and time T.

	MUSCULAR SYSTEM	MECHANICAL SYSTEM
FORCE LMT^{-2}	$M^{2/3}$	M^{1}
POWER OUTPUT L^2MT^{-3}	$M^{2/3}$	M^{1}
SPEED LT^{-1}	M^{0}	M^{0}
ACCELERATION LT^{-2}	$M^{-1/3}$	M^{0}
	$L = M^{1/3}$ $T = M^{1/3}$	$L = M^{0}$ $T = M^{0}$

Applying the same procedure to a mechanical system, L and T are obtained as M to be the power of zero. This means that mechanical systems of different sizes cannot be geometrically similar, and operate on the same time scale which is independent of their size.

In the mechanical system, the time scale seems to be applied as an independent parameter. This feature may be explained by the fact that the mechanical system is originally designed for human use, and thus it is designed to work according to the same time scale as that of man. Any machine, regardless its size, should be designed to have reasonable speed and acceleration. The robot, for example, is expected to work for humans with the same time scale as that of human life, while a physiological system does not work for humans but for itself.

To specify this feature of the mechanical system, the expression "homochronicity" may be adequate, just as the expression "homomorphism" specifies the features of the muscular system. In addition, the muscular system may be specified more clearly as homomorphic and heterochronic, since muscular systems of different sizes have different time scales. On the contrary, the mechanical system may be specified to be heteromorphic and homochronic, because each system is governed by the same time scale. Homomorphism and homochronicity cannot be realized simultaneously. Theoretically, a homochronic but heteromorphic muscular system can be conceived, but the nature did not design muscle in such a way. Nature tends to prefer homomorphism rather than homochronicity, even though this choice does not ensure maximum power output.

CONCLUSION

A comparative analysis of the mass exponents of fundamental parameters of muscular and mechanical systems showed an obvious difference between the two. Homomorphism is commonly recognized in the forms of muscular systems, while this feature is not realized in mechanical systems. On the other hand, the time scale of a muscular system depends on its size, while a mechanical system has a time scale which is independent of its size.

Although muscle and the cylinder within an engine can both be considered to be devices which convert chemical potential into mechanical power output, these two systems seem to be governed by different laws. One favours homomorphism rather than attaining a better energetic performance, while the other favours only energetic performance regardless of form.

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Q: I wonder if you have any idea about the reason why in biological structures "homomorphism" has been preferred.
(T. Takahashi)

A: It seems difficult to answer this question, although some explanation such as the convenience in morphogenesis may be thought. But no one can confirm it. So I dare say, it is a preference of nature.

C: It would appear that life span is used by nature as a variable parameter. It has been shown in Drosophila that by delaying the age of first mating the life span of the insect could be increased by 50% after relatively few generations. I think that this may be of relevance to some of your studies.
(V. Howard)