Science on Form: Proceedings of the First International Symposium for Science on Form,
General Editor: S. Ishizaka, Editors: Y. Kato, R. Takaki, and J. Toriwaki, 33-38.

Diffusion-limited Aggregation with a Restriction of Growth Direction

Hiroshi Kondo(1), Mitsugu Matsushita(1) and Shuhei Ohnishi(2)

(1) *Res. Inst. Electr. Commun., Tohoku University*
(2) *Exploratory Lab., NEC Fundamental Res. Labs.*

Keywords: DLA, fractal, anisotropy

A new type of two-dimensional diffusion-limited aggregation (DLA) is computer-simulated in which the growth direction is restricted within some angle θ. The seed particle from which the DLA cluster grows is located at the corner of a wedge-type boundary with the angle θ. Brownian particles released far from the DLA cluster are either killed (absorption-type boundary condition) or reflected (reflection-type boundary condition) when they reach on the wedge boundary. Average length L and width W are found to be scaled as $L \sim N^{\nu_{\parallel}}$ and $W \sim N^{\nu_{\perp}}$, where N is the total number of aggregated particles in the DLA cluster. The θ-dependence of the exponents $\nu_{\parallel}$ and $\nu_{\perp}$ and the ratio L/W are discussed.

INTRODUCTION

Diffusion-limited aggregation (hereafter abbreviated as DLA) model proposed by Witten & Sander (1981) is the simplest one of the growing fractal models. However, we do not understand sufficiently the reason why the DLA or other fractal patterns should be self-similar in nature. In this paper, we first consider the essential mechanism of the DLA. Next, we modify the original DLA model so as to change the fractal structure for further investigation of the aggregation process.

ORIGINAL DLA MODEL

The original DLA model is computer-simulated in the follwing way. Let us take two-dimensional lattice space for simplicity. First of all, a seed particles is put at the origin. A Brownian particle is launched randomly far from the seed (cluster), and steps to one of the nearest neighbor lattice points with equal probability. When the particle happens to arrive at the interface of the cluster, it joins the cluster as the new member. Then, another random walker starts off, and above procedure are repeated until the aggregation spreads large enough. Consequently, the DLA cluster is growing to an open-branched structure with the fractal dimension nearly 1.7 .

The algorithm of the DLA model shows that the fundamental mechanism of the cluster growth is constructed with the following conditions and processes :

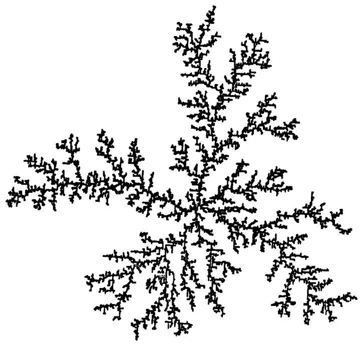


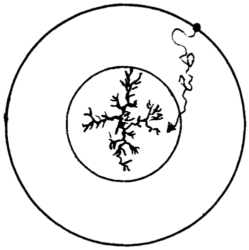
Fig. 1 Original DLA.

(1) The region where the cluster is grown is not restricted. So the cluster spreads outerwards (Fig. 1).
(2) The elementary diffusion process is isotropic.
(3) There is no preferential sticking direction to the cluster within the lattice bond direction. Then, the sticking mechanism is also isotropic.

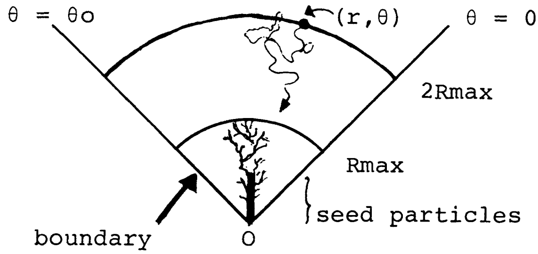
If one of these items is replaced by anisotropic one, the fractal structure of the DLA cluster may change. This modification is corresponding to the generalization of the DLA model. The generalized DLA is applicable to the analysis of the pattern formation phenomena in Nature and in the experimental systems. For exsample electrodeposition (Matsushita, 1984)(Brady, 1984), viscous fingering (Nittmann, 1985)(Ben-Jacob 1985), dielectric breakdown (Niemeter, 1984), and perhaps the drainage network of the river (Horton, 1945). These phenomena are dominated by diffusion process in different scales, but the geometrical structures are similar with each other.

MODIFICATION OF DLA MODEL AND ITS SIMULATION

Here, we treat the case of the restriction of the region where the Brownian particle diffuses and the DLA cluster is grown. Mathematically, the restriction of the growth area corresponds to the boundary condition, where the diffusion particles are absorbed or reflected. The shape of the boundary must be scale-invariant like an infinite line. We use wedge-shaped boundary with the open angle θo. (Fig. 2b). The seed particles for the aggregation are put on the corner of the wedge. Sinse the diffusion particle cannot come to the corner of the wedge easily when the size of the cluster is not so large, the seed particles are arrayed linearly. This does not affect the latter stage of the cluster growth at all. This is merely a technique of the computer-simulation.



(a)



(b)

Fig. 2 : (a) Normal(Original) DLA.
(b) DLA with restricted growth direction.

In the normal DLA, a Brownian particle is launched randomly from a position on a circle surrounding the cluster with equal probability (Fig. 2a). On the other hand, the probability distribution is not constant on the launching arc when the diffusion field is bounded. Then, one must calculate such a distribution function in order to simulate this new type of DLA model. Random walk yields the diffusion equation in the continuum limit :

$$\partial C/\partial t = D\ \nabla^2 C \qquad (1)$$

where C is the concentration of the diffusion particles. Since the cluster growth is very slow, the variation of C is very small in the far area from the cluster. Then the Laplace equation

$$\nabla^2\ C = 0 \qquad (2)$$

is the fundamental equation for the DLA model. In the case of the absorption-type boundary condition

$$C(r,\theta) = 0 \quad \text{for} \quad \theta = 0 \quad \text{and} \quad \theta = \theta o \qquad (3)$$

the solution becomes

$$C(r,\theta) \sim r^n \sin(\ n\theta\ )\ ; \quad n = \pi/\theta o \qquad (4)$$

Thus the probability distribution function $P(\theta)$ takes the form

$$P(\theta) \sim \sin(\ n\theta\ ) \qquad (r = \text{constant}) \qquad (5)$$

where (r,θ) is the launching position as shown in Fig. 2(b).

The radius of the launching arc is chosen 2Rmax , where Rmax is the maximum distance of the cluster from the origin. If the diffusion particle goes away from the cluster, it is killed and another particle is started off from the launching arc. The criterion of the judgement is the arc of radius 3Rmax . It is better for the accracy of the Monte Carlo simulation to enlarge these radii. Here we choose these values of the radii in order to reduce the execution time of the computer-simulation. In the case of the reflection-type boundary condition, the current of the diffusion particles is zero on the boundary, and we cannot speci-

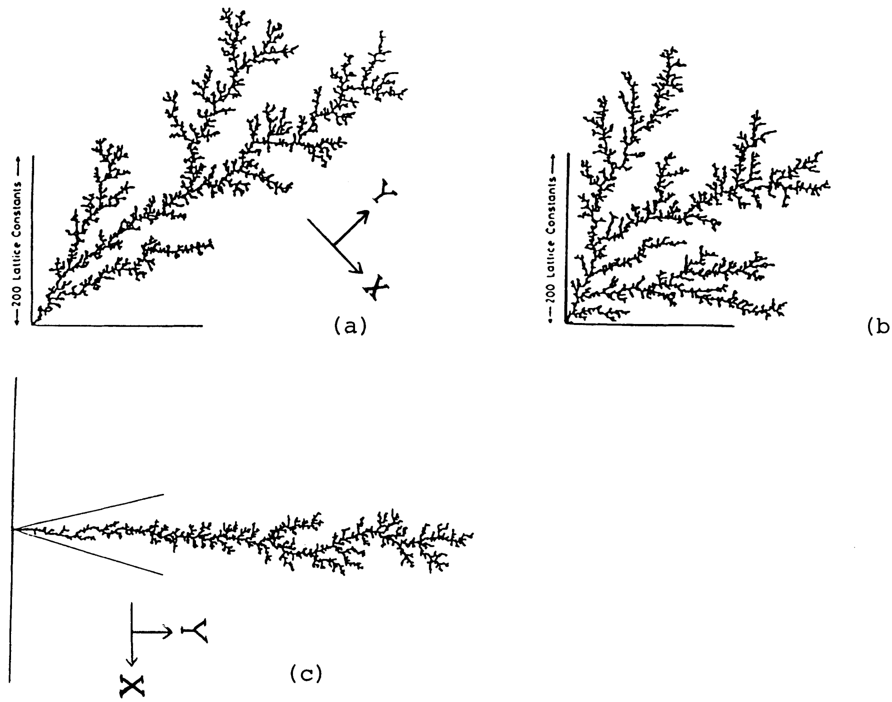


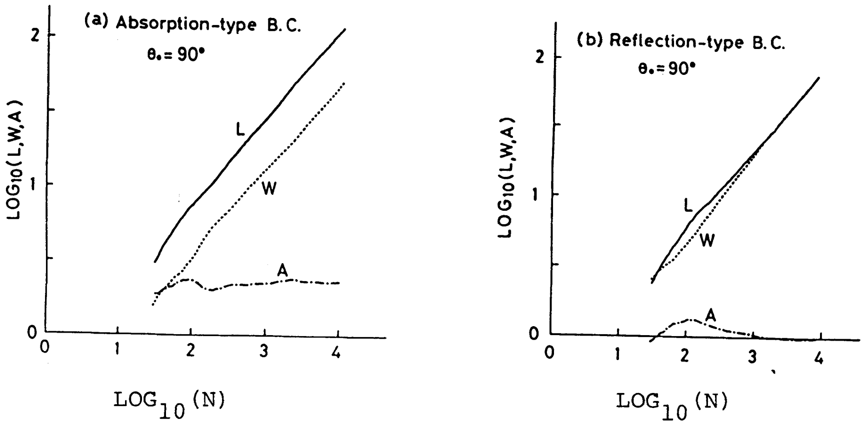
Fig. 3 Examples of modified DLA cluster.
(a) $\theta o = 90^\circ$ Absorption-type B.C.
(b) $\theta o = 90^\circ$ Reflection-type B.C.
(c) $\theta o = 30^\circ$ Absorption-type B.C.

fy the value of C. So we choose $P(\theta)$ = constant and the launching arc to be 3Rmax. The radius of the judgement of the kill is 5Rmax. The real $P(\theta)$ is determined heuristically. The DLA cluster growth in the case of this boundary condition is used only for the comparison with that of absorption-type one.

The patterns of the DLA cluster are shown in Fig. 3. The screening effect by the absorption-type boundary is obviously seen when comparing the patterns (3(a) and 3(b)) under differnt boundary conditions each other. The anisotropy can be expected in the DLA cluster growth, particularly when θo is small (Fig. 3(c)).

EVALUATION OF ANISOTROPY

In order to investigate the anisotropic growth process, we introduce $R_{\parallel}$ and $R_{\perp}$, which represent approximate length L and width W of the DLA cluster respectively. They are calculated by means of radius of gyration ;



(a) (b)

Fig. 4 Log-log plot of the dependence of $R_{\parallel}$, $R_{\perp}$, A on N.
$\theta o = 90$; $L = R_{\parallel}$, $W = R_{\perp}$.
(a) Absorption-type B.C.
(b) Reflection-type B.C.

$$L(N) \propto R_{\parallel}(N) = (\Sigma(Xi-Xc(N))^2 / N)^{1/2} \tag{6a}$$

$$W(N) \propto R_{\perp}(N) = (\Sigma(Yi-Yc(N))^2 / N)^{1/2} \tag{6b}$$

where (Xi,Yi) is the i-th sticking position of the particle, (Xc,Yc) is the center of mass, and N is the number of the particles constructing the cluster. Let us assume the scaling behavior of $R_{\parallel}$ and $R_{\perp}$, write them as

$$R_{\parallel} \sim N^{\nu_{\parallel}} \tag{7a}$$

$$R_{\perp} \sim N^{\nu_{\perp}} \tag{7b}$$

and express the aspect ratio with R and R by

$$A(N) \equiv R_{\parallel}(N) / R_{\perp}(N) = N^{\nu_{\parallel} - \nu_{\perp}} \tag{8}$$

If $\nu_{\parallel} = \nu_{\perp}$, we regard the growth process as truly anisotropic. The log-log plots of $R_{\parallel}$, $R_{\perp}$, A on N are shown in Fig. 4. Both of them are average of 10 trials of the simulation. The anisotropic behavior are not seen when $\theta o = 90^{\circ}$, since the exponents are almost of equal value. On the othe hand, in the case of $\theta o = 30^{\circ}$, the anisotropy emerges numerically. Fig. 5 shows the dependence of the exponents $\nu_{\parallel}$ and $\nu_{\perp}$ on θo. When $\theta o = 0$, $\nu_{\parallel} = 1$, $\nu_{\perp} = 0$, the anisotropy is complete. At a certain angle between 30° and 50° , $\nu_{\parallel}$ and $\nu_{\perp}$ become differnt values. This may be regarded as relating with the recently reported fact (Meakin, 1985) that in the normal DLA, decay exponent of the tangential density-density correlation function is different from that of radial one when the azimuthal angle is smaller than about 40°.

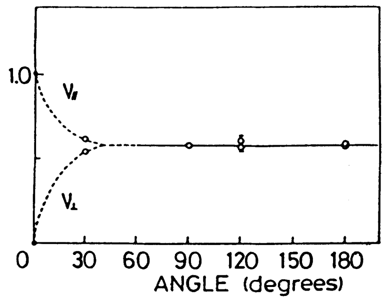


Fig. 5 The dependence of $\nu_{\parallel}$ and $\nu_{\perp}$ on θ_o.

CONCLUSION

In conclusion, by introducing the growth exponents $\nu_{\parallel}$ and $\nu_{\perp}$, the anisotropy is found to be emerge in the process of the DLA cluster when the open angle of the boundary that restricts the cluster growth is small. This indicates that the fractal structure of the DLA cluster varies in the growth process of parallel direction and perpendicular direction.

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